Stochastic Models

ISSN: 1532-6349 (Print) 1532-4214 (Online) Journal homepage: https://www.tandfonline.com/loi/lstm20

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To cite this article: Sándor Molnár & Lajos Vágó (2020): Networking in the absence of congestion control, Stochastic Models, DOI: 10.1080/15326349.2020.1742160

To link to this article: https://doi.org/10.1080/15326349.2020.1742160

Published online: 25 Mar 2020.

STOCHASTIC MODELS
https://doi.org/10.1080/15326349.2020.1742160

Networking in the absence of congestion control

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ABSTRACT

We study a future Internet networking paradigm where instead of congestion control an open loop traffic control is applied. We aim to give theoretical foundations for data transfer controlled only by the access points of the network. The key characteristics of networks without congestion control are stability and efficiency addressed in this paper. We consider the queue length processes of data-flows on directed graphs. The stability is characterized by the ergodicity of these processes and the efficiency of the network is measured by the Price of Anarchy. Under restrictions on the input traffic rates we derive an achievable efficiency limit in a stable network for very general conditions, namely, for any network topology and for any buffer management policy. Moreover, we show that even for cyclic networks, which usually cause severe instability in networks, an upper bound for the loss of efficiency can be given independently of the size of the network under a fair AQM buffer management policy. Furthermore, for monotonic networks we present a reasonable choice for setting access capacities. Our results demonstrate that with a proper setting of access capacities of incoming flows the congestion collapse of the Internet can be avoided even without congestion control.

ARTICLE HISTORY
Received 26 September 2017
Accepted 10 March 2020

KEYWORDS
Congestion control; ergodicity of queueing processes; open loop traffic control; TCP

1. Introduction

This paper investigates a communication network model where we use no closed loop control to appeal for traffic congestion. While the motivation comes from technology, we chose to investigate a specific stochastic model of the network without going much into the technological details and concentrate on the mathematical challenges raised by the topic. For completeness we give a brief summary of the motivation in the following paragraphs.

1.1. Motivation, historical background

There is a common belief that the stability of the Internet is mainly due to the *congestion control algorithm* performed by the *Transport Control*

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Protocol (TCP). The history of TCP dates back to 1981 when the first version of TCP was published[22]. Over the past four decades TCP has been step by step developed in order to meet the requirements of the continuously evolving Internet and to efficiently protect the network from congestion collapse[13]. It can be concluded based on the history of the Internet that the closed-loop congestion control implemented by TCP, which transports more than 85% of Internet traffic, was a successful paradigm to avoid congestion collapse and the related performance degradation due to the overload of network resources. The basic algorithm of the congestion control mechanism of TCP is that the sender gradually increases the sending rate until a packet loss is detected. In case of packet loss a feedback signal is sent back from the receiver to the sender indicating a possible congestion in the network. As a result, the sending rate is cut in half and the cycle repeats.

As researchers faced new challenges of the modern heterogeneous networking environments of the Internet a number of new TCP versions had been suggested[1,11,16,20,21]. In addition, several alternative ideas have also been proposed to find a transport protocol that provides good Quality of Experience (QoE) to the users. For example, Google introduced a new experimental protocol called Quick UDP Internet Connections (QUIC)[24] in 2013. QUIC uses User Datagram Protocol (UDP) in the transport layer instead of the traditional TCP and combines a collection of techniques to achieve better performance than TCP does. Google implemented QUIC in its popular Chrome browser. Another promising protocol called Bandwidth Bottleneck and Round (BBR)-trip propagation time[7] was proposed by Google in 2016. BBR was design to obtain an optimal operation point suggested by Kleinrock in 1979[14]. It can avoid bufferbloat and it seems to be an efficient protocol for achieving small buffer sizes. However, BBR is still under development (currently BBR v2) and regarding its application a lot of questions remained unanswered.

1.2. Related literature, mathematical background

A shocking idea of *networking without congestion control* first appeared in a research plan of Global Environment for Network Innovations (GENI)[8] in 2007. They proposed the omission of the congestion control mechanism and suggested to use efficient erasure coding to cope with congestion in the network. Some related works have only been published in this area as follows. Raghavan and Snoeren[23] have shown that even a congested network can achieve good performance and fairness. They have introduced a concept called decongestion control, presuming that a protocol relying upon greedy, high-speed transmission has the potential to perform better than TCP.

From the mathematical point of view the most relevant for this paper is the work of Bonald et al. and Feuillet[3,10]. They have studied the consequences of operating a network without congestion control, and have concluded that it does not inevitably lead to congestion collapse as believed earlier. In these papers the authors model a communication network with a directed graph with edge capacities and the data flowing through it follows a continuous time discrete space Markov process. They conjecture that if the underlying directed graph is acyclic then the asymptotic efficiency of the network is optimal as the individual access rate of the users to the network approaches 0. They verify this conjecture in the important special cases of tree and line network topologies. One of the key approaches they use is to show that in the above limit the stochastic process they investigate approaches a deterministic process called fluid limit.

There are a number of other suggestions applying efficient erasure coding (e.g., fountain codes[6,9,15,17,19]) instead of congestion control but the sound theoretical foundations of networking avoiding congestion control mechanisms have not been established yet. As a conclusion we have found that networking without congestion control is still a new and unexplored approach for future networks especially regarding basic theoretical foundations of the idea, and this is the main motivation behind our research.

1.3. Our contribution

Our work is close in the spirit to the work by Bonald et al.[3] and Feuillet[10] but our investigated architecture and model are different. These differences are discussed in the following section in details. Our ultimate goal is to contribute to the understanding of networks where congestion control is not applied and establish the theoretical foundations for such networks.

The main contribution of this paper is to present some theoretical results regarding the *stability* and *efficiency* of network models where congestion control is not applied. Concerning stability we study the ergodicity of the processes under investigations. As an efficiency measure we focus on the Price of Anarchy, which is a frequently used metric, also used in Bonald et al.[3] Most results are valid on the packet level (Theorems 3.1, 3.2 and 3.3, for the description of packet model see Sections 2.1 and 2.2), and one assumes fluid model (Theorem 3.4, see Section 2.5 for definition). Namely, on the packet level first we give a result (Theorem 3.1) for general topology and general buffer management policy for the case where the input rates are approximately specified. Then we investigate the circle network

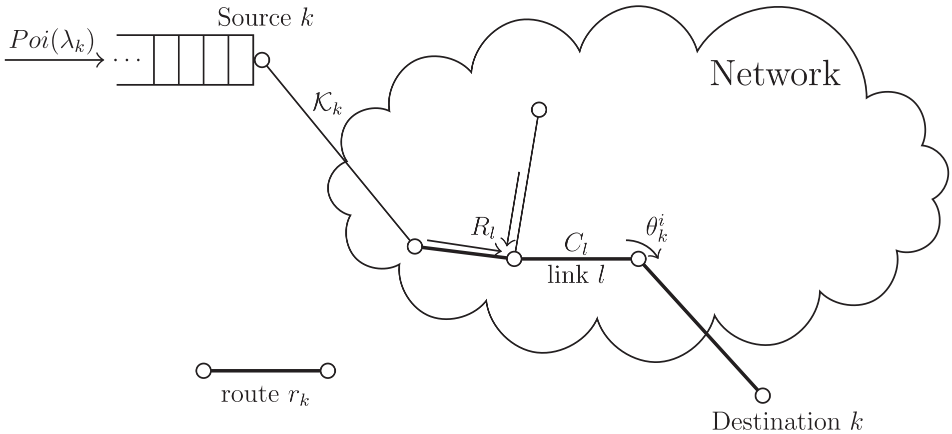


Figure 1. Network model. The notations are introduced in Section 2.1. The boldfaced edges are the links of route r_k. R_l is the offered input rate of link l, and θ_k^i is the throughput of flow k on the i-th segment of route r_k.

topology (Theorems 3.2 and 3.3), which is an interesting class of special network topology from stability point of view since circles usually cause instability in networks, and therefore represent worst-case network topologies[3]. In this case we assume a fair Active Queue Management (AQM)[12] in order to ensure the fair capacity distribution of the link between the competitive flows. It is a frequently applied method to do this job and it is supported by the current Internet routers since such mechanisms, e.g., Weighted Fair Queueing (WFQ)[27] or Deficit Round Robin (DRR)[25] are widely implemented in these routers. As results we show upper bounds for the achievable efficiency where stability can be held. In addition, we investigate acyclic and monotonic networks in the fluid model (Theorem 3.4) and show that these processes are stable under minimal conditions with the proper access capacity setting.

The rest of the paper is organized as follows: In Section 2 we introduce our network and traffic model and the most important notions. Then we present and discuss the main results of this paper in Section 3. The proofs are given in Section 4. Finally, Section 5 concludes the paper.

2. Network and traffic model

First we give a brief introduction to our network and traffic model, and then we introduce the necessary notations and work out the details in the following sections.

We consider flows on a given directed graph with given link capacities, see Figure 1 and notations are given in Section 2.1. We refer to this graph as the *base graph*. Note that we use the “flow” as a general entity in our model and it can be applied in practical cases like aggregate of flows, etc.

Note also that the flow in our proposed network has a different practical interpretation compared to what we have in our congestion controlled networks of today. For example, there is no similar meaning of the concept of streaming flows or elastic flows in our case. Flows join to the network through access links with given access capacities. In each flow packets arrive according to a Poisson process. We use the Poisson assumption only for analytic tractability, but we believe that the results could be generalized without this assumption as well, e.g., this assumption is not needed in Theorem 3.1. Packets of the same flow can queue up at the beginning of their path in an unbounded buffer, that is, no packet losses are assumed here. Packets are transmitted at maximum rate in each flow, i.e., with the rate equal to the capacity of its access link. Competing flows share flow rate according to some buffer management policy at each node of the network as described later in details.

We assume that there are no packet buffers considered before the subsequent links of the flows in the network nodes. Therefore, in this network if a flow has sent some data via the first few links of its path and after that it cannot reach enough flow rate on the next link, then the extra data gets lost. In order to cope with such packet loss, which are often called as dead packets, we assume an *erasure coding scheme accomplished by fountain codes*[18], e.g., the technique and our recently published transport protocol called Digital Fountain based Communication Protocol (DFCP)[19]. This digital fountain based method uses no congestion control and the lost packets are recovered by efficient Raptor codes[26]. The principle of this method is that the sender can transfer a theoretically infinite stream of encoded symbols from the original message of size k. Successful decoding can be performed with high probability as once any subset of size $\lceil (1+\varepsilon)k \rceil$ encoded symbols (here $\varepsilon > 0$ denotes the amount of redundancy added to the original message) arrive to the receiver. It is important to note that in realistic scenarios only slightly more packets are required for successful decoding than the original size of the message. It means that the price we pay for the open loop control in this solution is the overhead of the applied fountain code based erasure coding scheme, which can be smaller than 5% in practical cases[19]. By the application of this method the information for recovering the loss packets are already" coded" into the sent packets so we can avoid the traditional retransmission procedures for recovery of lost data. We mention that erasure coding scheme accomplished by fountain codes is just one possible solution to the mentioned dead packet problems caused by packet losses. Any other mechanism can also be applicable which makes flows robust to packet loss.

We emphasize that our approach is an *open-loop* control method in contrast to the *closed-loop* method of TCP congestion control. The only *traffic* *control* we apply in this network model is to put *access capacities* where flows are injected into the network, which can shape the maximum sending rate. The access rate can be set and policed e.g., by traffic shaping techniques, which is a widely known and supported functionality in current access points. However, our proposal addresses a new networking paradigm which is not necessarily based on the infrastructure of today's networks. Our goal is to find an access capacity setup which maximizes the throughput settings (in some sense as described later) such that the obtained queue length process in the access buffers is stable (ergodic).

A similar problem was investigated in Bonald et al. and Feuillet[3,10] but our model is different. In our model we consider a bufferless packet level network model where each flow has an access capacity to the network with an unlimited access buffer (outside the network).

The motivation of using this network model is that effects of buffers inside the network is rather unpredictable (e.g., bufferbloat) and these buffers are often difficult to control so if we have a network without such buffers we can get a more controllable and computable networking architecture. On the other hand, a policy function is usually implemented at the edge of the network so implementing access buffers is a reasonable assumption. In this paper we are setting up the theoretical foundations of such networks and we apply the unbounded access buffer assumption here as a first step. From a practical point of view the implementation of the access capacities can be easily managed by traffic shapers which are commonly used in today's networks.

We suppose Poisson packet arrivals to the access buffer resulting in a *Markovian model for the queue length dynamics* in the access buffer of each flow. In contrast, authors in Bonald et al. and Feuillet[3,10] considered a similar bufferless network model at flow level and in their model there are no access buffers and they modeled the *number of flows in progress as a Markovian process* as flows are generated by users and also cease upon completion. It also means a significant technical difference: in case of the model in Bonald et al. and Feuillet[3,10], as the number of flows tends to infinity their joint access rates grows to infinity as well, while in our model the access rate of each flow is fixed, no matter how big the number of queueing packets is.

Note that the Poisson assumption of the arrival process is a technical assumption which makes the mathematical calculations easier, but we believe that this assumption can be relaxed.

2.1. Notations

We use the following notations in the paper, see also Figure 1. Link l has capacity C_l (packet/sec). There are a total number of K flows in the network. Let us denote by λ_k (1/sec) the incoming Poissonian intensity of new packets of flow k and by $n_k(t)$ the number of queueing flow-k packets in the access buffer at time t. We denote by $r_k = \left(r_k^1, \ldots, r_k^{d_k}\right)$ the ordered list of links in the path of flow k, where d_k stands for the number of links in that path. In addition, each flow k is connected to the first node of the network by a link of access capacity $\mathcal{K}_k$.

2.2. Queue length process

We assume any fair AQM (Active Queue Management) policy as a buffer management mechanism in our network model at each node. For example, Weighted Fair Queueing (WFQ)[27] or Deficit Round Robin (DRR)[25] can be applied since they are used by Internet routers. Such AQM policies can fairly distribute the capacity of the link between the competitive flows.

According to this, the applied AQM works in such a way that competing flows are allowed to send an amount of data per second proportional to their demand at each link. This is described by formulas in the following way.

Let us denote by R_l the total (or offered) input rate of link l. In addition, let θ_k^i stand for the output rate of flow k on the i-th segment of its route, which is the same as the input rate of flow k on its $i+1$-th link. Put

$$\theta_k^0(t) = 1_{n_k(t)>0}(t)\mathcal{K}_k, \tag{1}$$

which is the input rate of flow k on its first link. That is, flows send data with the maximal bandwidth allowed by the access capacities whenever there is data in the queue. We often suppress the dependence on t in the notation. By definition

$$R_l = \sum_{k,\, i: r_k^i = l} \theta_k^{i-1}.$$

The output rate of flow k on the first and i-th link of its path due to the fair AQM policy are as follows:

$$\theta_k^i = \theta_k^{i-1} \min\left\{\frac{C_l}{R_l}, 1\right\} \text{ if } l = r_k^i, i = 1, \ldots d_k. \tag{2}$$

Note that (2) is the same as Equation (5) in Bonald et al.[3] and Equation (3) in Feuillet[10], which describes the effect of the Tail Dropping buffer management policy on the sharing of the link resources among the competing users (the" users" are packets in this packet-level model, flows in Bonald et al. and Feuillet[3,10]). Bonald et al.[3] showed that transmission rates θ_k^i are uniquely defined by (2). Let us denote by ψ_k the throughput of flow k:

$$\psi_k = \theta_k^{d_k}, \tag{3}$$

and by $\boldsymbol{\psi}$ we denote the throughput vector:

$$\boldsymbol{\psi} = \boldsymbol{\psi}(\mathbf{n}) = (\psi_1, ..., \psi_K).$$

We note that the throughput $\boldsymbol{\psi} = \boldsymbol{\psi}(t)$ depends only on the non-empty queues regardless of their queue lengths.

Let $\mathbf{n}(t) = (n_1(t), ..., n_K(t))$ be the queue length process at the sources. With the above input and service rates it is a continuous time Markov process with transition rates

$$\begin{cases} \lambda_k : & \mathbf{n} \to \mathbf{n} + \mathbf{e}_k, \\ \psi_k : & \mathbf{n} \to \mathbf{n} - \mathbf{e}_k, \end{cases}$$

where $\mathbf{e}_k$ stands for the vector with 1 in its k-th coordinate and 0 in the others.

To determine the throughput of the flows at a given time we only need to know which flows have positive, and which have zero queueing data, so the exact number of packets does not matter. Hence, from the point of view of the throughput there are 2^K different states. We call these the *queue length indicator* and denote by $\mathbf{I}(t) \in \{0, 1\}^K$, where the k-th coordinate of $\mathbf{I}(t)$ is 0 if $n_k(t) = 0$, and it is 1 if $n_k(t) > 0$:

$$I_k(t) = 1_{n_k(t)>0}.$$

Therefore we can consider θ_k^i and ψ_k as functions from $\{0, 1\}^K$ to $\mathbb{R}_{+,0}^K$, where $\mathbb{R}_{+,0} = [0, \infty)$.

2.3. Networks with partially decreasing throughput function

Later in Theorem 3.4 (and also in the proofs of Theorem 3.2 and 3.3) we will use a monotonicity property defined in the following way.

Definition 2.1. *We say that the throughput vector $\boldsymbol{\psi}$ is partially decreasing if for all $\mathbf{x} = (x_1, ..., x_K), \mathbf{y} = (y_1, ..., y_K) \in \{0, 1\}^K$, such that $x_i = y_i$ and for all j $x_j \leq y_j$, we have*

$$\psi_i(\mathbf{x}) \geq \psi_i(\mathbf{y}).$$

Among many others, dumbbell topologies and directed trees are monotonic networks in the above sense. On the other hand, the parking lot topology is an example of an acyclic but non-monotonic network, see Figure 2.

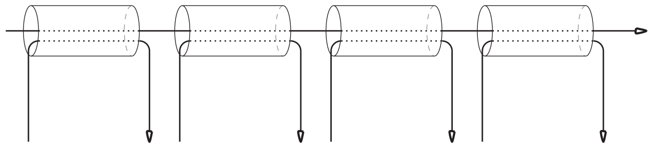


Figure 2. Parking lot topology with $L = 4$ links and 5 flows. For $L \geq 2$ links this network is non-monotonic: If there is queuing data in the first short flow (leftmost) then this flow blocks the long flow, hence lets more flow rate to the second short flow.

2.4. Stability and efficiency

We say that a network with given input rates and access capacities is *stable* if $\mathbf{n}(t)$ form an ergodic process, or equivalently any queue empties in finite expected time.

Given a network we say that the input rates $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_K)$ satisfy the *optimal stability condition* if

$$\forall l \sum_{k:l \in r_k} \lambda_k < C_l. \tag{4}$$

This is the biggest set of input rates for which one can get ergodicity with an optimal traffic control, and for example for α-fair allocations it is also sufficient[4]. However, our congestion control is suboptimal, i.e., there may be packets sent to the network which eventually get lost due to capacity limitations, so-called dead packets. These packets occupy resources in the network, so from efficiency point of view this is waste of resources. In addition, as (4) is the optimal stability condition implied by the inside links of the network, a non-optimal traffic control can unnecessarily limit traffic flows at the access points which may also result in loss of efficiency. Because of these suboptimalities the set of stable input rates can be smaller than the optimal one, see Figure 3.

Let us fix some set of the *admissible input rates* $D \subset \mathbb{R}^K_{+,0}$ such that if $\mathbf{u} \in D$, then for all $c > 0$ we have $c\mathbf{u} \in D$ as well. For a given network and fixed access capacities $\mathcal{K}_1, \ldots, \mathcal{K}_K$ we define the *Price of Anarchy* as in Bonald et al.[3], which quantifies the efficiency loss caused by omitting closed loop control and letting the users to follow a greedy tactic and inject as much data to the network as they can. It is defined by

$$P_D(\mathcal{K}_1, \ldots, \mathcal{K}_K) = \max_{\mathbf{u} \in D} \left(\frac{\alpha(\mathbf{u}) - \beta(\mathbf{u})}{\alpha(\mathbf{u})} \right), \tag{5}$$

where $\alpha(\mathbf{u})$ is the supremum of those α-s for which $\alpha\mathbf{u}$ satisfies the optimal stability condition, and $\beta(\mathbf{u})$ is the supremum of those β-s for which $\beta\mathbf{u}$ is stable according to the traffic control determined by the access capacities and by a buffer management policy (e.g., fair AQM), see Figure 3.

Then we denote by P_D the least loss of efficiency which can be reached by optimizing access capacities, that is

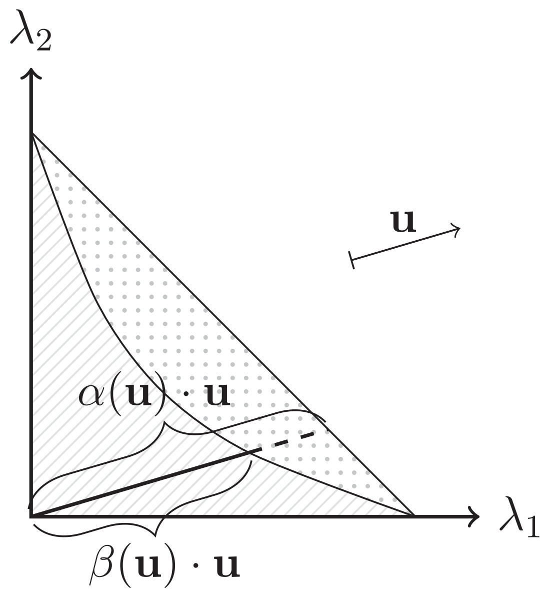


Figure 3. An example of stability and optimal stability regions of two flows. The striped area refers to the stability region, while the union of striped and dotted areas is the optimal stability region.

$$P_D = \inf_{\mathcal{K}_1,\dots,\mathcal{K}_K} P_D(\mathcal{K}_1,\dots,\mathcal{K}_K).$$

Clearly $\alpha(\mathbf{u}) \geq \beta(\mathbf{u})$ for any access capacities because stability with some input rates under a fair AQM policy immediately implies stability under optimal control as well. Hence $P_D \in [0,1]$, if $P_D = 0$ it means that the network has maximum, 100% efficiency. Note that the Price of Anarchy is typically defined in such a way that the range of possible values is $[1,\infty]$, but instead we chose to use the notations of Bonald et al.[3]. In the following we always consider *efficiency* determined by $1 - P_D$. Note that there are two opposite mechanisms that may cause efficiency loss: On the one hand, if control capacities are too high then dead packets may waste capacity, and on the other hand, if access capacities are unnecessarily strict then in some cases the capacity of the network is not fully exploited.

2.5. Fluid model

In this paper we also study a deterministic fluid model $\{\mathbf{x}(t)\}$ related to the above random process of queue lengths. We define the dynamics of $\{\mathbf{x}(t)\}$ – similarly to Feuillet[10] [Theorem 3, (24)] – by the differential equation system

$$\dot{x}_k(t) = \lambda_k - \psi_k(\mathbf{x}(t)) \tag{6}$$

for all k, where ψ_k is the throughput of flow k defined almost the same way as $\psi_k(\mathbf{n}(t))$. We define by (2) and (3) the throughput of flow k on later edges of its path, but applied for x instead of n. But it is necessary to define the input rate on the first edge in a slightly different way when $x_k(t) = 0$. This is simply because the values of $\boldsymbol{\psi}(\mathbf{x}(t))$ when $x_k(t) > 0$ for all k define the values of it when $x_k(t) = 0$ for some coordinates k. Therefore, instead of (1) we define $\theta_k^0(t)$ by

$$\theta_k^0(t) = \begin{cases} \mathcal{K}_k \text{ if } x_k(t) > 0, \\ \theta_k^*(\mathbf{x}(t)) \text{ if } x_k(t) = 0, \end{cases}$$

where $\theta_k^*(\mathbf{x}(t))$ is maximal such that $\psi_k(\mathbf{x}(t)) \le \lambda_k$ and $\theta_k^*(\mathbf{x}(t)) \le \mathcal{K}_k$.

Note that λ_k and ψ_k are just the *average* input and throughput rates, respectively, therefore the $\{\mathbf{x}(t)\}$ process is the mean field approximation of $\{\mathbf{n}(t)\}$. Also note that by the definition the throughput vector $\boldsymbol{\psi}(\mathbf{x}(t))$ depends only on which elements of $\mathbf{x}(t)$ are nonzero.

If the optimal stability conditions (4) are satisfied, then $\mathbf{x} \equiv 0$ is clearly a fixed point of the fluid process with $\theta_k^*(0) = \boldsymbol{\lambda}$ and no congestion. In Theorem 3.4, we provide conditions under which the fluid process converges to this fixed point 0.

3. Main results on stability and efficiency

In this section we state and discuss our main results regarding the stability and efficiency of networks without congestion control. The proofs are in Section 4. The results are divided into three parts. In the first one we consider the packet-level model with general topology and general buffer management policy for the case where the input rates are restricted to a narrow domain D (as in (5)). In this case stability is guaranteed under mild conditions (Theorem 3.1). As the assumption is quite restrictive, we consider this result as a reference and exclude this assumption in the following results.

The second part serves as a worst case example. We investigate the packet model with circle network topologies (Theorems 3.2 and 3.3), which usually cause instability in networks. This is the part where we rely both on the Poisson packet-arrival model and the fair AQM policy. We use the Poisson assumption for analytic tractability only and we believe that the results on circle networks hold without it as well, but we did not prove it here. Although the proofs of Theorems 3.2 and 3.3 strongly rely on the symmetry of the circle topology and therefore can't be directly generalized, we consider these results very relevant because they emphasize two important and complementing facts: (i) we can't expect perfect efficiency from every network with the given control method; and (ii) even a cyclic network can be controlled with a limited efficiency loss. In addition, we also consider cycles as a basic building block representing the effect of feedback, which also motivates its investigation.

Finally, we investigate acyclic and monotonic networks in the third part. Here, we consider the fluid model for analytic tractability. We obtain the stability of queue length processes in such models under weak, necessary conditions (Theorem 3.4).

Note that our results are not built on one another, the order in which we present them is just a matter of taste and does not reflect their importance.

3.1. General network topology

First we consider general network topology, i.e., the underlying directed graph with edge capacities on which the flows transmit data is arbitrary. We give an upper bound on the Price of Anarchy if the set of admissible input rates form a cone. Let us use the notation $\mathbb{R}_+ = (0, \infty)$ and $\mathbb{R}_{+,0} = [0, \infty)$. For $\mathbf{u} \in \mathbb{R}_+^K$ and $c \geq 0$ let $D_c(\mathbf{u}) \subset \mathbb{R}_+^K$ be defined by

$$D_c(\mathbf{u}) = \left\{ \boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_K) \in \mathbb{R}_+^K \;\middle|\; \forall i, j = 1, \dots, K : \frac{1}{1+c} \frac{u_i}{u_j} \leq \frac{\lambda_i}{\lambda_j} \leq (1+c) \frac{u_i}{u_j} \right\}, \tag{7}$$

see Figure 4. That is, the ratios of the input rates of flows are restricted to a neighborhood of prescribed rate ratios.

Theorem 3.1. *Let* $\mathbf{u} \in \mathbb{R}_+^K$*, i.e.,* $\forall\, i u_i > 0$*. For any network topology and for any buffer management policy we have*

$$P_{D_c(\mathbf{u})} \leq \frac{c}{1+c},$$

which is reached by the access capacity setting

$$\mathcal{K}_k = \alpha(\mathbf{u}) u_k,$$

where $\alpha(\mathbf{u})$ *is defined in (5). We note that* $\alpha(\mathbf{u})\mathbf{u}$ *is the boundary point of the optimal stability region in direction* $\mathbf{u}$.

This theorem has practical significance if c is close to 0, i.e., the aspect ratio of the cone is small (see Figure 4), hence the loss of efficiency is close to 0 as well. We also note that if c is small we have a strong bound on the ratios λ_i/λ_j of the input rates regardless of the rates themselves.

The above access capacity setting ensures that there is no congestion at all in the network. From the engineering point of view, this theorem shows how efficiently we can allocate resources. The strength of this theorem is that it holds for any network topology and for any buffer management policy. Moreover, even the Markov condition on the input process can be

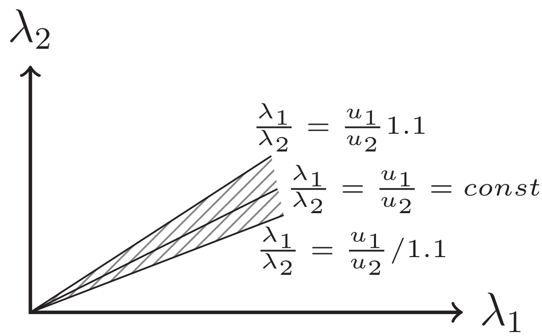


Figure 4. The striped area is an example of the cone $D_c(u)$ for two flows, $u = (2, 1)$ and $c = 0.1$.

omitted. It is enough to suppose that incoming data comes from a stationary process.

In the following the circle network topology is investigated, which represents a *worst-case network topology* from the point of view of stability[10].

3.2. Circle network topology

We consider a circle network structure. Let the graph be a directed circle of $K \geq 2$ links, with the links labeled by $1, \ldots, K$ according to the orientation of the graph. We have K flows on this graph each going through two consecutive links, i.e., $r_k = (k, k+1), k = 1, \ldots, K-1, r_K = (K, 1)$, see Figure 5. The optimal stability condition (4) is satisfied iff

$$\lambda_k + \lambda_{k+1} < C_{k+1} \quad \forall k = 1, \ldots, K-1 \text{ and } \lambda_K + \lambda_1 < C_1. \tag{8}$$

For the Price of Anarchy of this network we prove the followings.

Theorem 3.2. *In case of the circle network with $K = 2$ flows and fair AQM policy, the followings hold for the Price of Anarchy:*

1. *The worst case is when $C_1 = C_2$, for which*

$$P := P_{\mathbb{R}^K_{+,0}} = 3 - 2\sqrt{2} \approx 0.1716,$$

which is achieved by the access capacity setting

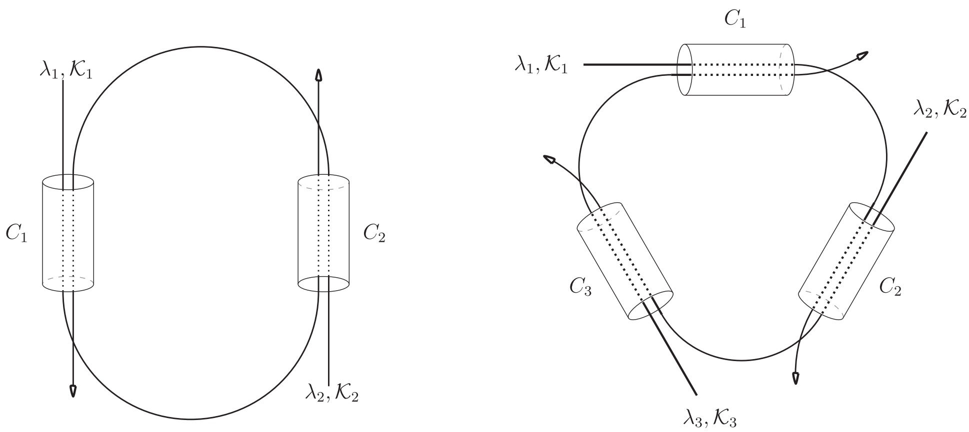
$$\mathcal{K}_1 = \mathcal{K}_2 = C_1\left(2\sqrt{2} - 2\right).$$

2. *If $C_1 \neq C_2$* then *the efficiency increases:*

$$P \leq 3 - 2\sqrt{2} \approx 0.1716, \tag{9}$$

which is achieved by

$$\mathcal{K}_1 = \mathcal{K}_2 = \min\{C_1, C_2\}\left(2\sqrt{2} - 2\right).$$



(a) A circle of 2 flows. (b) A circle of 3 flows.

Figure 5. Circle networks. Tubes represent links, lines with arrows are the flows.

3. *Moreover, if* $3/2C_1 \le C_2$*, then the efficiency is maximum, i.e.,*

$$P = 0,$$

reached by the access capacity setting $\mathcal{K}_1 = \mathcal{K}_2 = C_1$.

Point (1) of Theorem 3.2 shows that when the two links have the same capacity, then for any access capacity settings the stability region is strictly smaller than the optimal one. Nevertheless, the Price of Anarchy (and hence the efficiency) is acceptable and the best access capacity setting is explicitly given in the theorem.

Point (2) of Theorem 3.2 shows that when the two capacities are different then the efficiency increases, and again the best access capacity setting is explicitly given.

Moreover, according to point (3) of Theorem 3.2 if $3/2C_1 \le C_2$, then the stability region coincides with the optimal one and hence maximum efficiency is achieved with the given access capacities.

Note that in Theorem 3.2 our goal is to find a universal access capacity setting without the knowledge of the actual input rates and regardless of the ratio of λ_1/λ_2. Equivalently to the statement, we can say that if the access capacities are set as above, then in order to achieve stable queues we must limit the input rates to $\left(1 - \left(3 - 2\sqrt{2}\right)\right)\boldsymbol{\lambda} \approx 0.8284\boldsymbol{\lambda}$ where $\boldsymbol{\lambda}$ satisfies the optimal stability condition.

We also remark that loss of efficiency may occur because of two phenomena. One is the existence of so-called dead packets, i.e., in case of the circle network with two flows it can happen that both flows use bigger flow rate on their first link than they are able to use on the second one. It means that they start sending data which gets lost before arriving to the destination. This waste of resources can lead to loss of efficiency. The other one is the flow rate limit forced by the access capacities even in the case if there was no congestion without them. For example, in case of point (1) of Theorem 3.2 if we set the access capacities as suggested by this theorem, then there can be dead packets and unnecessary access capacity limitations as well (as it turns out from the proof). By increasing access capacities the number of dead packets increases as well so that the efficiency decreases altogether. On the other hand, by decreasing access capacities the impact of limitations of access capacities are more severe than that of the decrease of dead packets resulting in a decrease of efficiency.

In the following theorem we consider the general circle topology with K flows and links as in Figure 5b, where $K = 3$.

Theorem 3.3. *In case of the circle network with $K \geq 3$ flows, $C = C_1 = \ldots = C_K$ and fair AQM policy, for the Price of Anarchy we have the following bounds independently of the number of flows.*

$$0.1716 \approx 3 - 2\sqrt{2} \leq P := P_{\mathbb{R}^K_{+,0}} \leq 0.262. \tag{10}$$

The upper bound holds for the access capacities

$$\mathcal{K}_k = C\sqrt{\left(9 + \sqrt{33}\right)/24} \approx 0.784C.$$

This theorem shows that for a circle of arbitrary number of links with the same capacities the efficiency $(1 - P)$ can be given in a 10% accuracy range, i.e., it is between 74% and 83%. Note that these bounds are independent of the number of flows K.

We have to remark that in case of the flow-level models investigated in Bonald et al.[3], the authors showed that the above structure leads to congestion collapse if the tail dropping policy is applied whenever all the input rates are positive, therefore $P = 1$. Please note, that this is not the case in our packet-level model, which is very promising from the application point of view. More precisely, the *undesired effect of circle networks can be stabilized* by introducing access capacities as suggested in our model. Therefore these common structures do not have to be excluded from networks where no congestion control is applied. Note that under another policy called fair dropping (see Bonald et al.[3, p.3]) there is no problem with the stability of circles even in flow-level models: In Bonald et al.[3], it was shown that if fair dropping is applied the optimal stability condition is sufficient for stability for any network topology, in particular for cycles.

3.3. Fluid model analysis of acyclic and monotonic networks

The previous results are quite restrictive either for the average input rate (Theorem 3.1) or the network topology (Theorems 3.2 and 3.3). To obtain results for more general topology and arbitrary input rates we investigate a somewhat simpler network dynamics, namely the fluid process $\{\mathbf{x}(t)\}$ defined in Section 2.5.

We say that the network is *acyclic* if the defining base graph contains no directed circles. In addition to this, we also assume that the throughput vector is *partially decreasing*, see Definition 2.1.

Theorem 3.4. *Suppose we are given an acyclic network topology with edge capacities C_l, $l = 1, \ldots, L$ such that the throughput vector $\boldsymbol{\psi}$ is partially decreasing. Suppose also that the optimal stability conditions (4) are satisfied for the input rates $\boldsymbol{\lambda}$. Set the access capacities $\mathcal{K}_k = \min\{C_l \mid l \in r_k\}$ for all flows k. Then the fluid process $\{\mathbf{x}(t)\}$ defined by (6) converges to the zero vector $\mathbf{0}$, regardless to the starting point $\mathbf{x}(0)$.*

That is, no matter how big the" queue lengths" $\mathbf{x}(0)$ at beginning are, the process $\{\mathbf{x}(t)\}$ stabilizes and converges to 0. In case of temporary queue length dynamics resulting in big queues congestion may appear. Nevertheless, according to the theorem above, queue lengths eventually smooth out within a finite time. In contrast with circle networks discussed in the previous section, in case of an acyclic and monotonic network congestion does not lead to a decrease of efficiency in the fluid model. This theorem also gives us a simple guideline to set up proper access capacities keeping the network stable.

4. Proofs

4.1. General network topology

Proof of Theorem 3.1. The main idea of the proof is to show that if the input rates are distributed according to the coordinates of $\mathbf{u}$, then we can reach optimal stability. This is achieved by simply partitioning the capacities of the links among the flows. Then it is not so hard to see that input rates close to $\mathbf{u}$ are near optimal as well.

Namely, choose the access capacity setting

$$\mathcal{K}_k = \alpha(\mathbf{u})u_k.$$

By definition $\alpha(\mathbf{u})\mathbf{u}$ is a bordering point of the optimal stability region. Therefore, according to the optimal stability condition (4), for all link l we have

$$\sum_{k:l\in r_k} \mathcal{K}_k \leq C_l.$$

Thus there is no congestion and no lost packets if we choose this access capacity setting. Hence, the queue length processes evolve independently of each other, that is each flow k has $\mathcal{K}_k$ flow rate independently from the other queue lengths. Moreover, if for $\gamma > 0$ the network with input rates $\gamma\mathbf{u}$ is stable under the optimal conditions, or equivalently $\gamma < \alpha(\mathbf{u})$, then all queue length processes are also stable if the network is controlled by the access capacities, under any buffer management policy. In other words, $\beta(\mathbf{u}) = \alpha(\mathbf{u})$, where $\beta(\mathbf{u})$ is defined in (5).

In the followings we show that this access capacity setting fits the other input rates of $D_c(\mathbf{u})$ (defined in (7)) as well in the sense of Theorem 3.1.

Let $\boldsymbol{\lambda} \in D_c(\mathbf{u})$. Without any loss a generality we assume that $\|\mathbf{u}\|_2 = \|\boldsymbol{\lambda}\|_2 = 1$ and that

$$\frac{\lambda_1}{u_1} \le \frac{\lambda_2}{u_2} \le \ldots \le \frac{\lambda_K}{u_K}. \tag{11}$$

Let

$$\alpha^* := \alpha(\mathbf{u})\frac{u_1}{\lambda_1} \text{ and } \beta^* := \alpha(\mathbf{u})\frac{u_K}{\lambda_K}, \tag{12}$$

see Figure 6. Then $\alpha^*\boldsymbol{\lambda}$ is not optimally stable. For contradiction, suppose it is. By definition $\alpha^*\lambda_1 = \alpha(\mathbf{u})u_1$, and

$$\alpha^*\lambda_k = \alpha^*\lambda_1\frac{\lambda_k}{\lambda_1} \ge \alpha(\mathbf{u})u_1\frac{u_k}{u_1} = \alpha(\mathbf{u})u_k$$

for all $k = 2, \ldots, K$, where we used (11) in the inequality. Then, since $\alpha(\mathbf{u})\mathbf{u} \le \alpha^*\boldsymbol{\lambda}$ coordinate-wise, hence the optimal stability of $\alpha^*\boldsymbol{\lambda}$ implies the optimal stability of $\alpha(\mathbf{u})\mathbf{u}$, which is contradiction (since the strict inequality in (4) implies that the" upper" border of the optimal stability region is open).

Similarly, by (11) and (12) we have $\beta^*\lambda_k \le \alpha(\mathbf{u})u_k$ for all $k = 1, \ldots, K$. Since $\alpha(\mathbf{u})\mathbf{u}$ is a bordering point of the *stability* region, hence, using that the queues evolve independently, we obtain that $\beta^*\lambda$ is stable, or a bordering point of the stability region.

Putting the above observations together we have

$$\alpha(\boldsymbol{\lambda}) \le \alpha^* \text{ and } \beta(\boldsymbol{\lambda}) \ge \beta^* \ \forall \lambda \in D_c(\mathbf{u}).$$

Hence

$$P_{D_c(\mathbf{u})} \le \frac{\alpha^* - \beta^*}{\alpha^*} = 1 - \frac{\lambda_K/u_K}{\lambda_1/u_1} = 1 - \frac{\lambda_K/\lambda_1}{u_K/u_1} \le 1 - \frac{1}{1+c},$$

where in the last inequality we used that $\boldsymbol{\lambda} \in D_c(\mathbf{u})$. □

4.2. Circle network topology

We start with some preliminaries. The proofs in this Section very much rely on the monotonicity property defined in Def. 2.1. For networks of partially decreasing and bounded throughput vector Borst et al.[5] described a

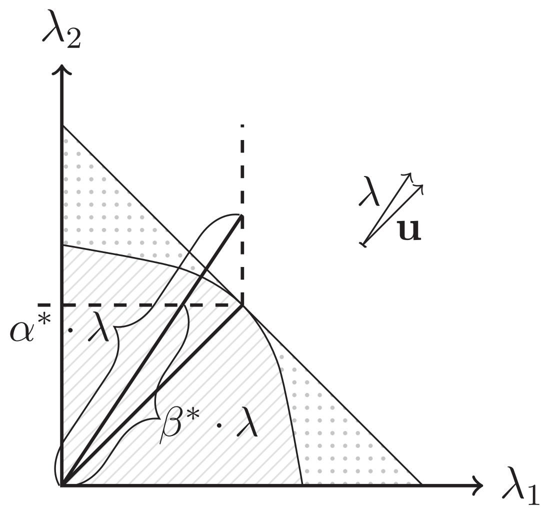


Figure 6. An example with two flows. The striped area refers to the stability region, while the union of striped and dotted areas is the optimal stability region. For the border of the optimal stability region we have the upper bound $\alpha^*\boldsymbol{\lambda}$ in direction $\boldsymbol{\lambda}$, and for the border of the stability region we have the lower bound $\beta^*\boldsymbol{\lambda}$.

sharp condition for stability. It turns out that to show stability the first thing we have to do is to find a queue which is stable even if the queue lengths of other queues tend to infinity (this is the worst case because of the monotonicity), and then to determine its stationary distribution under this condition. After that we have to find another stable queue under the condition that the length of the first queue is in its stationary distribution while the others are infinitely long. Now again these two stable queue lengths have a unique joint stationary distribution. Then we have to find the next stable queue and so on until there aren't any. If we are able to show the stability of all queues one by one in this way, then we conclude that the whole queue length process is stable. Moreover, this criterion is necessary as well in the sense of Theorem 3, p.13. of Borst et al.[5].

To formalize the above algorithm precisely we introduce some notations. Let

$$L_i^k(\lambda_1, \ldots, \lambda_k; \psi) := \sum_{\mathbf{x} \in \{0,1\}^k} \psi_i(I_1, \ldots, I_k, 1, \ldots, 1)\pi^k(\mathbf{I}),$$

where $\pi^k(\mathbf{I})$ stands for the stationary distribution of the queue length indicator of the first k queues under the assumption that the last $K - k$ queues are non-empty. We use the following theorem.

Theorem 4.1 (Theorem 2, p.11 of Borst et al.[5]). *Let* $(n_1, \ldots, n_K)$ *be the queue length process with arrival rates* $\lambda_1, \ldots, \lambda_K$ *and bounded partially* *decreasing throughput vector* $\psi = (\psi_1, \ldots, \psi_K)$*. Assume that there exists* k *such that*

$$\lambda_i < L_i^{i-1}(\lambda_1, \ldots, \lambda_{i-1}; \psi)$$

for all $i = 1, \ldots, k$*. Then the processes* $n_1, \ldots, n_k$ *are stable, regardless of the initial state.*

Proof of Theorem 3.2. First suppose $C_1 = C_2$. We suppose without loss of generality that $C_1 = C_2 = 1$. The symmetry of network topology suggests to search for equal access capacities:

$$\mathcal{K}_1 = \mathcal{K}_2 =: \mathcal{K}.$$

Suppose $1/2 \leq \mathcal{K} \leq 1$.

First, we determine the throughputs of flows in all the possible 4 states $\mathbf{I} \in \{0,1\}^2$ of the queue length indicator. By symmetry we have to distinguish only 2 types of states depending on the number of nonempty queues: when both queues are non-empty, and when only one of the flows has queueing data. If only one of them is nonempty, say flow 1, then $\psi_1(1,0) = \mathcal{K}$. Else if both are nonempty, then the (common) throughput of the flows are obtained from the following equations: recall that we denoted by $\theta_k^1 = \theta_k^1(1,1)$ and by $\theta_k^2 = \theta_k^2(1,1)$ the (mutual) output rates on the first and on the second link on the way of the flow, respectively. Then we have

$$\begin{cases} \theta_k^2 = \frac{\theta_k^1}{\theta_k^1 + \mathcal{K}}, \\ \theta_k^1 + \theta_k^2 = 1. \end{cases} \tag{13}$$

The first equation follows from the fair AQM policy, and the second one holds because the links are saturated whenever both flows send data and $\mathcal{K} \geq 1/2$. The solution of these equations is clearly $\psi_k(1,1) = \theta_k^2 = \left(2 + \mathcal{K} - \sqrt{4\mathcal{K} + \mathcal{K}^2}\right)/2$.

Set

$$\mathcal{K} = 2\sqrt{2} - 2. \tag{14}$$

Then

$$\psi_1(1,0) = 2\sqrt{2} - 2 \text{ and } \psi_{\mathrm{k}}(1,1) = \sqrt{2} - 1. \tag{15}$$

Note that this is the choice for which the total throughput of two flows is the same as the throughput of one flow when the other queue is empty. We will later see that the Price of Anarchy is minimal with this $\mathcal{K}$ choice. Once the throughputs are known one can obviously conclude the following:

Lemma 4.2. □ *The throughput vector* $\psi = (\psi_1, \psi_2)$ *is partially decreasing.*

First we prove that $P < 3 - 2\sqrt{2}$ cannot be achieved for any access capacities. If the input rates are proportional to $\mathbf{u} = (1, 0)$, i.e., when only flow 1 has incoming packages, the Price of Anarchy is clearly $(1 - \mathcal{K})/1$, which increases while reducing $\mathcal{K}$.

On the other hand, consider the case $\mathbf{u} = (1/2, 1/2)$, i.e., the input rates of the flows are equal. If both queues are nonempty then according to the solution of (13) we can guarantee that they empty iff for their input rate vector $\boldsymbol{\lambda}$ we have $\lambda_1 = \lambda_2 < \psi_k(1, 1)$. Therefore the Price of Anarchy in this case is $\left(1/2 - \psi_k(1,1)\right)/(1/2) = 1 - \mathcal{K}$ as before, compared to the border $(1/2, 1/2)$ of the optimality region. In addition, since $\psi_k(1, 1)$ decreases if we increase $\mathcal{K}$, hence this loss increases while increasing $\mathcal{K}$. So we have proved that one cannot hope $P < 3 - 2\sqrt{2}$.

Second, we show that the Price of Anarchy does not increase if we consider other cases as well. Suppose that a vector $\mathbf{u}$ satisfies the optimal stability (8). Suppose $u_1 \leq u_2$. Our aim is to show that for $(\lambda_1, \lambda_2) = \mathcal{K}\mathbf{u} = \left(2\sqrt{2} - 2\right)\mathbf{u}$ stability holds under fair AQM policy. To this end, we use Theorem 4.1. We claim that flow 1 is stable under the worst case assumption that flow 2 always has data in its buffer. For contradiction, suppose that $\lambda_1 \geq \psi_k(1, 1)$. Then using (14) and (15) we get

$$u_1 + u_2 \geq 2u_1 = 2\lambda_1/\mathcal{K} \geq 2\psi_k(1,1)/\mathcal{K} = 1,$$

which contradicts the optimal stability condition. Hence flow 1 is stable under the condition that flow 2 is nonempty. Moreover

$$\pi^1(1) = \lambda_1/\psi_k(1,1) \text{ and } \pi^1(0) = 1 - \lambda_1/\psi_k(1,1).$$

Then flow 2 is also stable while the queue length indicator of flow 1 is in its stationary distribution:

$$L_2^1(\lambda_1; \psi) = \frac{\lambda_1}{\psi_k(1,1)}\psi_k(1,1) + \left(1 - \frac{\lambda_1}{\psi_k(1,1)}\right)$$
$$\mathcal{K} = \lambda_1 + \left(1 - \frac{\lambda_1}{\psi_k(1,1)}\right)2\psi_k(1,1) = \mathcal{K} - \lambda_1 > \lambda_2,$$

where in the last inequality we used that, according to the optimal stability of $\mathbf{u} = \boldsymbol{\lambda}/\mathcal{K}$, we have $\lambda_1 + \lambda_2 < \mathcal{K}$.

There remains the case $3/2C_1 \leq C_2$. Suppose $C_1 = 1$. Setting $\mathcal{K}_1 = \mathcal{K}_2 = 1$, it is easy to see that when both queues are nonempty, for the throughput rates we have

$$\theta_1^1(1,1) = \frac{1}{2}, \theta_1^2(1,1) = \frac{1}{2}, \theta_2^1(1,1) = 1 \text{ and } \theta_2^2(1,1) = \frac{1}{2}.$$

Otherwise, when exactly one flow has data to send, then it has flow rate 1, which, similarly to the above calculations, implies stability under the optimal conditions $\lambda_1 + \lambda_2 < 1$. □

Before turning our attention to the proof of Theorem 3.3 we note that circles with 4 or more flows are not partially decreasing in general. For example, let $K=4$ and $\mathcal{K} = C = 1$. Then it is easy to see that from the point of view of the first flow it is better if flow 3 (which is disjoint of flow 1) is nonempty, because it makes bottleneck for flow 2 and flow 4, which are competing with flow 1. Numerically, after straightforward computations we have

$$\psi_1(1,1,1,1) = 1 - \frac{-1+\sqrt{5}}{2} \approx 0.382$$

and

$$\psi_1(1,1,0,1) = \frac{0.5}{0.5+1} = \frac{1}{3},$$

which contradicts to the partially decreasing property. In the following proof we estimate the throughput vector with a partially decreasing one which enables us to use Theorem 4.1.

Proof of Theorem 3.3. Again, the symmetry of network topology suggests to search for equal access capacities:

$$\mathcal{K}_1 = \ldots = \mathcal{K}_K =: \mathcal{K}.$$

Without loss of generality we suppose $C=1$. Since the flows use two links, hence one can show that $P \geq 3 - 2\sqrt{2}$ with the setting $\mathcal{K} = 2\sqrt{2} - 2$ in the same way as in the proof of Theorem 3.2.

Now let us give an upper bound on the Price of Anarchy. Suppose $\mathcal{K} \geq 1/2$. Our aim is to show that if for the input rates we have

$$\lambda_k + \lambda_{k+1} < 0.738 \;\; \forall k = 1, \ldots, K-1 \;\text{ and }\; \lambda_K + \lambda_1 < 0.738,$$

then the circle network is stable if $\mathcal{K}$ is chosen appropriately. We show that the queue length process of flow 1 is ergodic, which is, by symmetry, equivalent to the ergodicity of all queues. We do it in such a way that we define another queue length process on the three flows: flow 1, flow 2 and flow K (i.e., flow 1 and its neighbors) with the same input rates but estimated throughput vector. We will see that the new, estimated throughput vector is partially decreasing, and the queue lengths of the processes stochastically dominate the original ones. Therefore, the ergodicity of the queue length of flow 1 under the new settings implies the ergodicity in the original case.

Let the first coordinate of the estimated throughput vector $\tilde{\boldsymbol{\psi}}$ be defined by

$$\begin{aligned}\tilde{\psi}_1(1,0,I_3,\dots,I_{K-1},0) &= \mathcal{K},\\ \tilde{\psi}_1(1,1,I_3,\dots,I_{K-1},0) &= \tilde{\psi}_1(1,0,I_3,\dots,I_{K-1},1)\\ &= \frac{\mathcal{K}}{\mathcal{K}+\mathcal{K}} = \frac{1}{2}, \text{ and}\\ \tilde{\psi}_1(1,1,I_3,\dots,I_{K-1},1) &= \frac{1/2}{1/2+\mathcal{K}},\end{aligned} \tag{16}$$

where $I_3,\dots,I_{K-1}$ can be either 0 or 1. It is easy to see that for any state $\mathbf{I}$ we have $\tilde{\psi}_1(\mathbf{I}) \le \psi_1(\mathbf{I})$. In addition, since $\tilde{\boldsymbol{\psi}}$ is partially decreasing, hence we give a lower bound on the total throughput of flow 1 if we estimate the throughput of flow 2 and flow K from below by

$$\tilde{\psi}_2(I_1,1,I_3,\dots,I_K) = \tilde{\psi}_K(I_1,\dots,I_{K-1},1) = \frac{1/2}{1/2+\mathcal{K}},$$

where $I_1,\dots,I_K$ can be either 0 or 1.

Moreover, due to the partially decreasing property, we can suppose that the input rates of flow 2 and flow K equal $\lambda := \max\{\lambda_2,\lambda_K\}$. We focus on the *estimated process* of flow 1, flow 2 and flow K defined by the new throughput vector $\tilde{\boldsymbol{\psi}}$ and input rates $\lambda_1,\lambda,\lambda$, and our goal is to determine the set of those parameters λ_1,λ for which flow 1 is ergodic.

Let

$$\mu(\lambda) := \sup\{\lambda_1 \mid \begin{array}{l}\text{the queue length of flow 1 of the}\\ \text{estimated process is stable}\\ \text{if the input rates of flows } 1,2,K\\ \text{are } \lambda_1,\lambda,\lambda, \text{ respectively.}\}\end{array}$$

Our goal is to show that for any $0 \le \lambda \le 0.738$ we have $\mu(\lambda)+\lambda \ge 0.738$, which immediately implies that for $\lambda_1+\lambda < 0.738$ the queue length of flow 1 of the estimated process is ergodic.

By the definition of the estimated throughput rates, the queue lengths of flows 2 and K evolve independently of each other and of flow 1. Note that they not always form an ergodic process. Namely, if $\lambda < \frac{1/2}{1/2+\mathcal{K}}$, then the stationary distribution exists and it is denoted by $\{\pi(I,J)\}_{(I,J)=0}^{1}$, where I and J are the queue length indicators of flow 2 and flow K, respectively. Clearly $\{\pi(I,J)\}_{(I,J)=0}^{1}$ is the product of the stationary distributions of flow 2 and flow K, since they are independent. Otherwise, if $\lambda \ge \frac{1/2}{1/2+\mathcal{K}}$, then we can suppose $\pi(1,1)=1$ because of the partially decreasing property.

Theorem 4.1 yields

$$\mu(\lambda)=\sum_{I,J=0}^{1}\pi(I,J)\tilde{\psi}_1(1,I,I_3,\ldots,I_{K-1},J).$$

If $\lambda<\frac{1/2}{1/2+\mathcal{K}}$, then by (16)

$$\begin{aligned}\mu(\lambda)=&\left(1-\frac{\lambda(1/2+\mathcal{K})}{1/2}\right)^2\mathcal{K}\\&+2\frac{\lambda(1/2+\mathcal{K})}{1/2}\left(1-\frac{\lambda(1/2+\mathcal{K})}{1/2}\right)\frac{1}{2}\\&+\left(\frac{\lambda(1/2+\mathcal{K})}{1/2}\right)^2\frac{1/2}{1/2+\mathcal{K}}.\end{aligned}$$

If $\frac{1/2}{1/2+\mathcal{K}}\leq\lambda\leq 1$, then we get

$$\mu(\lambda)=\frac{1/2}{1/2+\mathcal{K}}.$$

Setting $\mathcal{K}=\sqrt{\left(9+\sqrt{33}\right)/24}\approx 0.784$, for any input rate $0\leq\lambda<1$ we have $\mu(\lambda)+\lambda\geq 0.738$. □

4.3. Fluid model

The sketch of the proof of Theorem 3.4 is quite simple. First we show that even if all the flows have positive queuing data some of them will get more bandwidth than their input rate. Moreover, thanks to the monotonicity property in Definition 2.1 we see that these queue lengths indeed converge monotonically to 0, which helps to show that other queues are going to become empty as well.

Proof of Theorem 3.4. We start with two basic observations:

- Setting up control links conserves the partially decreasing property and does not introduce circles, and
- The control links defined in Theorem 3.4 does not decrease the optimally stable region.

The worst case scenario from the point of view of a flow is when all the other flows have "queueing data", $x_k(t)>0$ for all k, let us assume this. If at some time $T\geq 0$ there is no saturated link in the network, then because of $\mathcal{K}_k>\lambda_k$ we have $\psi_k(t)=\mathcal{K}_k>\lambda_k$ for all $t>T$ as long as $x_k(t)>0$, that is all queues decrease to 0 linearly. Moreover, because of the partially decreasing property saturation cannot occur after a queue empties, therefore all $x_k(t)$ converges to 0.

Now suppose that there is saturation on some links at the beginning. As the network is acyclic, there is a numbering of the links (including control links) such that each flow follows a path of links with increasing number. Let us fix such a numbering. We show that there are flows getting enough flow rate even if $x_k(t) > 0$ for all k. As there are saturated links, there is a last one among them at time $t_0 = 0$ (the link with highest number among the saturated ones, l_0, say). Now consider the set F_0 of flows going through l_0. From the optimal stability condition (4) follows that

$$\sum_{k \in F_0} \psi_k(\mathbf{x}(t_0)) = C_{l_0} > \sum_{k \in F_0} \lambda_k,$$

and thus there are $k \in F_0$ such that $\psi_k(\mathbf{x}(0)) > \lambda_k$. Flows transmitting enough data in this worst case empty linearly. Let us denote by F_1 the set of flows k which get enough bandwidth when all queues are positive, that is

$$F_1 = \{k \mid \psi_k(\mathbf{x} = 1) > \lambda_k\}.$$

F_1 is nonempty by the above argument. By the monotonicity property, queue lengths of flows in F_1 converge to 0, let t_1 be such that for $t \geq t_1$ $x_k(t) = 0$ for all $k \in F_1$. For these flows for $t \geq t_1$ the final throughput is $\psi_k(\mathbf{x}(t)) = \lambda_k$. Due to saturation of links these flows might use more bandwidth than λ_k on early links of their path, but on the last saturating edge and after that on nonsaturating ones its exactly λ_k. Then we can go on with the proof by induction: For $t \geq t_1$ if there is no saturation, then all flows empty linearly as we argued before, or if there is a last saturating link, then for some flows k outside F_1 we have $\psi_k(\mathbf{x}(t)) > \lambda_k$ and these flows will empty, and so on. □

5. Conclusion

We investigated a network traffic model to contribute to the theoretical foundations of a future Internet without congestion control, focusing on the stability and efficiency characteristics. We found that if input rates can vary in a cone then the loss of efficiency (Price of Anarchy) can be bounded by the size of the cone with explicitly determined access capacity settings. The importance of this result from a practical point of view is that it shows the limit of achievable efficiency in a stable network operating without any congestion control for very general conditions, namely, for any network topology and for any buffer management policy.

For cyclic networks with fair AQM buffer management policy we obtained upper bounds for loss of efficiency (Price of Anarchy), which is independent of the size of the network. The result is interesting enough since circles are in general the worst network topologies causing instability in networks, for example, in the model of Bonald et al.[3], the loss of efficiency (Price of Anarchy) is 100% for circles.

Finally, in order to design proper access capacities of the network from the point of view of the stability of the network, we studied the relationship of stability and access capacities in the fluid model. We proved that in acyclic networks if the partially decreasing property holds then the network can easily be stabilized with the proper access capacity setting. In addition, the setting that we use in Theorem 3.4 is explicitly given and is the smallest one that one could use without decreasing the stability region as compared to the optimal region.

As our future work we intend to generalize our fluid flow results for non-monotonic networks. Our goal is to analyze arbitrary networks composed by both monotonic network structures like trees and non-monotonic network elements like circles. Circle networks and other non-monotonic blocks composed with other elements may not be optimally efficient, therefore analyzing the performance of such constructions could be the aim of future simulations. Furthermore, we work on omitting the Markovian assumptions of the queue length dynamics to obtain general results, as we believe that the Markovian property is only a technical assumption.

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