A General Fractal Model of Internet Traffic

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Abstract

The fractal nature of Internet traffic has been observed by several measurements and statistical studies. In this paper a new monofractal stochastic process called Limit of the Integrated Superposition of Diffusion processes with Linear differential Generator (LISDLG) is presented, which effectively characterizes network traffic monofractality. Several properties of the LISDLG model are presented including covariance structure, cumulants, spectrum and bispectrum. The model captures high-order statistics by means of the cumulants. The relevance and validation of the proposed model are demonstrated by application studies for measured Internet traffic.

1 Introduction

A number of studies in the last decade confirmed the presence of scaling behavior of network traffic [14, 24, 4, 16]. Significant research was carried out to obtain a good understanding of these observed fractal-like phenomena. Several studies showed the presence of monofractal scaling at large-time scales (from a few hundreds of milliseconds and above) which can be modeled by self-similar models [14, 12, 4, 5, 9]. However, some studies indicate that in small time scales a more complex scaling property can be observed which is consistent with multifractal scaling [4, 5, 9]. Some recent studies argue that this multifractal scaling may be present even at large-time scales [13].

The analysis, characterization and modeling of monofractal scaling captured by self-similar models (e.g. on/off models, Cox's M/G/∞ models, Fractional Brownian Motion models, FARIMA models, etc.) have been well established in teletraffic research during the previous decade [24]. However, the physical explanation of these phenomena still represents open research issues. Some studies show that the on/off dynamics of traffic flows with heavy-tailed file sizes can result in self-similar behavior [3, 18]. Other studies argue that the TCP can adapt to self-similar fluctuations and propagate it to other parts of the Internet where there is no physical reason for its generation [22].

In contrast to self-similar modeling the observed fractal phenomena are still poorly understood. It becomes clear that self-similar traffic models which capture first and second order statistical characteristics result in an incomplete description of network traffic. Higher-order statistical characteristics should also be taken into account for an accurate traffic model. This means that multifractal models seem to be more relevant in the case of traffic with significant non-trivial high-order statistics. However, if we build these additional information into our traffic models we might increase the complexity of the model, so these models are recommended from a practical point of view only in the cases where there are significant effects of high-order statistics on the performance metrics. These complex models can also be useful if we can get a deeper understanding of network traffic dynamics by using these multifractal models.

The physical explanations of multifractal scaling properties which appear in network traffic are rather incomplete at this time. Recent studies explain this behavior by observing that networks appear to act as conservative cascades [4]. The motivation of this conjecture is based on the fragmentation process of TCP/IP protocol hierarchy. Other explanations say that the multifractality at small time scales is due to the TCP flow control [6]. Other studies argue that the interactions between network elements resulting in a complex buffering and multiplexing effects are responsible for the multifractal behaviour [15]

The complex fractal nature of Internet traffic has been observed by several measurements and statistical studies starting with Taqqu et al. [23] and followed by a number of studies [4, 5, 9, 15, 13]. However, so far only a few relevant multifractal models have been developed and these are based on constructing a multiplicative process structure

[4, 7].

In this paper a new monofractal model is presented which is not self-similar and has a more powerful modeling ability to capture scaling behavior compared to self-similar models. We argue that our monofractal model is flexible enough to accurately capture the fractal scaling of network traffic and no need to use more complex multifractal models. In this model we fit the cumulants of the measured traffic to the cumulants of the process generated by the model and show that the resulted bispectrum of the traffic can be accurately captured.

The paper is organized as follows. Our measurements are described in Section 2. We present the mathematical background of cumulants, high-order spectra, mono- and multifractality in Section 3. Our proposed new monofractal model with its properties is described in Section 4. The application of the model for our measured Internet traffic is presented in Section 5. Finally, the paper is concluded by Section 6 with a summary of our main results.

2 Traffic Measurements

Our measurements were gathered in a university campus network shown in Figure 1. As a part of the University Network a number of LANs located at the Informatics Building of the Budapest University of Technology and Economics (BUTE) are connected to the outside world by a 100MB FDDI and a 155MB ATM link via the University backbone network. In the figure DG denotes a Department Group and each DG is built on an Ethernet based LAN and consists of about 100 workstations. These workstations belong to staff members, PhD students, and student laboratories using a variety of operating systems and network interfaces ranging from 10Base2 (BNC) through 100BaseT (UTP) to 100VGAnyLAN. Connections between DGs and between a DG and the outer world are guaranteed by the ATM backbone. Ethernet frames are transmitted over the ATM backbone using LAN emulation. Traffic generated by LANs and going to the global Internet are multiplexed by the FORE ES-2810 ATM switch and passed to the router.

Our measurements are based on some outstanding features of the FORE ES-2810 instrument. This ATM switch has the ability to make an on-line monitoring of packet flows arriving at its input ports. We chose the most loaded link for measurements. The switch was configured to copy the chosen traffic flow to another port which was routed to a Red Hat Linux workstation configured only for our measurement goals. All packets were captured by the *tcpdump* [1] utility. We carried out our measurements continuously during a nearly two week period from 4 to 16 November 2000.

[1]Tcpdump is available at *http://www-nrg.ee.lbl.gov/*

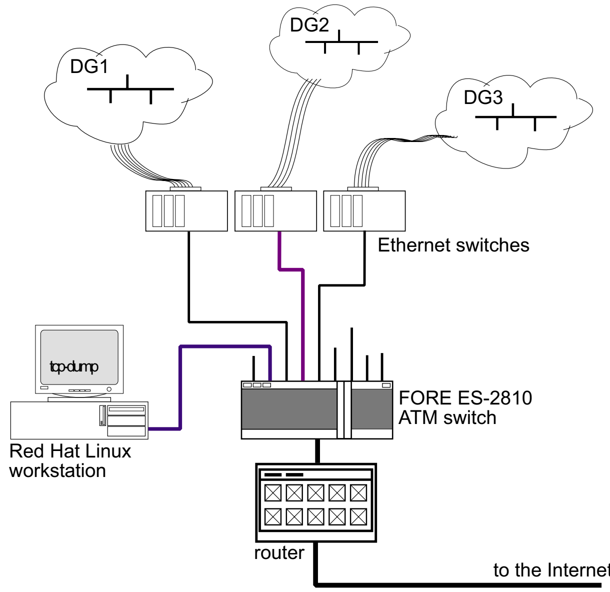

Figure 1. The configuration of the traffic measurements

3 Preliminaries

Here we give a brief account of the analysis of data in frequency domain.

3.1 Cumulants, spectra and estimation of bispectrum

For properties of cumulants and higher order spectra we refer to the books [1], [17] and [20]. If a strictly stationary process $Y(t)$ has third order moments, then not only the covariances $\mathrm{Cov}(Y(t), Y(t+s))$ are invariant with respect to the time-shift but the third order cumulants $\mathrm{Cum}(Y(t), Y(t+r), Y(t+s))$ as well. The third order cumulants are just the third order central moments, i.e. $\mathrm{Cum}(Y(t), Y(t+r), Y(t+s)) =$
$\mathsf{E}\left(Y(t) - \mathsf{E}Y(t)\right)\left(Y(t+r) - \mathsf{E}Y(t+r)\right)$
$\left(Y(t+s) - \mathsf{E}Y(t+s)\right)$ but in general they are the coefficients of the Taylor expansion of the log-characterisitic function. The cumulants of order m can be expressed by the central moments of order less than or equal with m. For instant the first six cumulants are given by the $C_\ell(Y) \doteq \mathsf{E}\left(Y - \mathsf{E}Y\right)^\ell$, i.e., the central moments of order

ℓ, as

$$\begin{aligned} \mathrm{Cum}_1(Y) &= \mathsf{E}Y \\ \mathrm{Cum}_2(Y) &= C_2(Y) \\ \mathrm{Cum}_3(Y) &= C_3(Y) \\ \mathrm{Cum}_4(Y) &= C_4(Y) - 3C_2(Y) \\ \mathrm{Cum}_5(Y) &= C_5(Y) - 10C_2(Y)C_3(Y) \\ \mathrm{Cum}_6(Y) &= C_6(Y) - 15C_4(Y)C_2(Y) - 10C_3(Y)^2 \\ &\quad + 30C_2(Y)^3. \end{aligned} \tag{1}$$

It is convenient to use the notations $z = e^{i\omega},\ z_k = e^{i\omega_k}$. The Fourier transform of the third order cumulants $S_{3,Y}$ is called the *bispectrum,*

$$S_{3,Y}(z_1, z_2) = \sum_{r,s=-\infty}^{\infty} \mathrm{Cum}(Y(0), Y(r), Y(s)) z_1^{-r} z_2^{-s}.$$

While the spectrum S_2 is real and nonnegative the bispectrum $S_{3,Y}$ is complex valued and it has the following properties of symmetry,

$$\begin{gathered} S_{3,Y}(z_1, z_2) = S_{3,Y}(z_2, z_1) = S_{3,Y}(z_1, z_3) = \\ S_{3,Y}(z_3, z_1) = S_{3,Y}(z_3, z_2) = S_{3,Y}(z_2, z_3) = \\ S^*_{3,Y}\left(z_1^{-1}, z_2^{-1}\right), \end{gathered}$$

where $z_3 = (z_1 z_2)^{-1}$, and * denotes complex conjugation. These equations imply that there are twelve triangles of frequencies in the plane, each of which can be considered as the basic domain for the bispectrum because it is completely specified over the entire plane if it is determined over one of the twelve triangles. We shall fix the triangle with vertices $(0,0), (\pi,0), (2\pi/3, 2\pi/3)$ as the basic domain for the bispectrum.

3.2 Self-similarity and Fractality

A wide variety of physical systems including data networks traffic exhibit fractal properties. We are interested in fractal data, i.e. data "look the same across of wide range of scales", at least in some regard. The notion of fractals, in the sense that it has similarity on all scales is translated into the stochastic analysis by the definition of *Self-Similar processes with Stationary Increments*. We shall denote it by H-SSSI. The stochastic process $Y(t)$ is called H-SSSI, if it has stationary increments and for all $a > 0$ real numbers

$$Y(at) \stackrel{d}{=} a^H Y(t),$$

where $\stackrel{d}{=}$ means the equality of the finite dimensional distributions. The parameter H is referred to as Hurst parameter or Hurst exponent. An example is the Fractional Brownian Motion (FBM) $B^{(H)}(t)$, where the parameter $H \in (0,1)$. Suppose that the m^{th} order cumulant, $\mathrm{cum}_m(Y(t))$ exists. Then it follows form the self-similarity that

$$\mathrm{cum}_m(Y(t)) = t^{mH}\, \mathrm{cum}_m(Y(1)),$$

i.e. $\log|\mathrm{cum}_m(Y(t))|$ scales linearly with respect to $\log(t)$ with coefficient mH. This property is referred to usually as monofractality, which comes down to the increments as well. A stationary process $X(k)$ is *multifractal* if

$$\log\left|\mathrm{cum}_m\left(X^{(n)}(k)\right)\right| = \beta(m)\log(n) + c(m), \tag{2}$$

where $\beta(m)$ is some (possible nonlinear) function of m and the aggregated series $X^{(n)}(k)$ is defined by

$$X^{(n)}(k) = \frac{1}{n}\sum_{j=0}^{n-1} X(kn - j), \quad k \in \mathbb{N}.$$

In general the subclass of multifractal processes for which $\beta(m)$ is linear will be called *monofractal*. If $X(k)$ is the series of increments of a H-SSSI process then $\beta(m) = m[H-1]$ i.e. it is linear according to m and the process is monofractal. Another particular case of multifractality is when $\beta(m)$ is constant. In that case it can not be increments of any H-SSSI process threfore the class of monofractal processes are wider then increments of H-SSSI processes. An example for such a monofractal process is the dilatative stable process, see [10] for details. Note that Taqqu [19] considers absolute moments instead of cumulants for the aggregated processes. As far as monofractals are concerned, the two definitions are equivalent. We prefer cumulants, because the scaling properties should not change with additive constants and summing up independent processes.

A number of studies showed that Poisson-like traffic models do not account for time dependencies observed at multiple time scales in network traffic, see [24]. This dependence structure in the time series is exhibited by the property that the variance of the aggregated series $\mathrm{var}(X^{(n)}(k))$ converge to zero slower than the rate n^{-1}. This property is usually called *Long-Range Dependence* (LRD) of network traffic. Let $r_k \doteq \mathrm{cov}(X(i), X(i+k))/\sigma^2$ denote the autocorrelation function. Recall that a stationary series $X(k)$ is Long-Range Dependent (LRD) if

$$r_k = L(k)k^{2H-2}, \quad k \to \infty,$$

where $L(k)$ is slowly varying at ∞,
i.e., $\lim_{x\to\infty}(L(tx)/L(x)) = 1$, for all $t > 0$.

It was observed that traffic on Internet networks exhibits the same characteristics regardless of the number of simultaneous sessions on a given physical link. At the same time the following characteristics were pointed out, see [15].

- Many signals possess significant LRD, but display short term correlations and behavior inconsistent with strict self-similarity.

- In many signals, the scaling behavior of moments as the signal is aggregated is a nontrivial (nonlinear) function of the moment order.

- Many signals have increments that are inherently positive and hence non-Gaussian.

There is an additional property which is motivated by our experimental study of ATM traces, see [21], providing strong evidence of presence of Gamma distribution and real valued bispectrum.

- Marginal distribution of many signals of ATM traces is close to Gamma.

- Many signals of ATM traces have real valued bispectrum

Several papers address the question of a more accurate description of Internet traffic, a broader model class, namely that of multifractal processes, has to be considered, see [19]. In our paper we propose a particular multifractal process, which is in the subclass of monofractal processes but not in the subclass of self-similar processes for network traffic description.

4 A Monofractal Model

The basic idea of the model comes from a particular construction of a fractal process as a limit, see [10] for detailes. The construction starts with the basic process of a superposition. It is called *Diffusion with Linear differential Generator* (DLG). It is the solution of the diffusion-type stochastic differential equation

$$dR(t) = (\mu + 2\alpha R(t))\,dt + 2\sigma\sqrt{R(t)}dB(t) \qquad (3)$$

where $\mu > 0$, $\alpha \in \mathbb{R}$, $\sigma > 0$ are the parameters. The solution $R(t)$ of the stochastic differential equation (3) is treated under the name: Diffusion process with a Linear differential Generator (DLG process). Note here that the DLG process has recently been used successfully for modeling interest rates, because it is one of the simplest models avoiding negative values [2]. It is referred to as CIR (Cox-Ingersoll-Ross) process and in the special case $\alpha = 0$ it would be called the squared Bessel process. Because of the form (3) it is called square-root diffusion, too. The results obtained by Watanabe and his coauthors show that there is a strong connection between the DLG process, infinitely decomposable diffusion processes and continuous-state branching processes [11]. It is also pointed out that the finite-dimensional distribution of the DLG process is multivariate Γ. The first step of the construction is a triangular array of random coefficient DLG processes, i.e. a sequence of series of independent stationary DLG processes with properly chosen random parameters. The row sums converge in a certain sense to a limit process Y_λ, called *Superposition of DLG processes* (SDLG), i.e.,

$$Z_n \mathrel{\hat{=}} \sum_{k=1}^{n} R_{n,k} \underset{n\to\infty}{\longrightarrow} Y_\lambda.$$

The SDLG process also has a Γ finite-dimensional distribution. It is stationary, it has a real-valued positive bispectrum and it is LRD. The main result is the existence of the *Limit of Integrated SDLG processes*, referred to as the LISDLG process.

$$J(t) \mathrel{\hat{=}} \lim_{\lambda\to 0} \int_0^t (Y_\lambda(s) - \mathrm{E}\, Y_\lambda)\, ds,$$

The fact is that there exists an a.s. continuous process J in $\mathcal{C}[0,T]$ with the following basic properties:

- The LISDLG process $J(t)$ has cumulants

  $$\mathrm{cum}_1(J(t)) = 0,$$

  and

  $$\mathrm{cum}_m(J(t)) =$$

  $$(m-1)!\frac{2^{1-2H}}{1-H} c_0 \sigma_0^{2m-2} \int_{[0,1]^m} D_{\tau_0}(\underline{s})^{2(H-1)} d\underline{s}\, t^{m+2(H-1)},$$

  for $m \geq 2$, where $H \in (1/2, 1)$,

  $$D_\tau(\underline{t}) \mathrel{\hat{=}} |t_{i_1} - t_1| + |t_{i_2} - t_{i_1}| \cdots + |t_1 - t_{i_{m-1}}|,$$

  $\tau = (i_1, \ldots, i_{m-1}) \in \mathrm{Perm}(2, 3, \ldots, m)$, $\tau_0 = (2, 3, \ldots, m)$. For $m = 1$, $D_\tau(t) \mathrel{\hat{=}} 0$.

- The LISDLG process $J(t)$ has the same covariance structure as that of the Fractional Brownian Motion (FBM). Namely, for $t_1, t_2 > 0$,

  $$\mathrm{cov}(J(t_1), J(t_2)) = \text{const.}\left(t_1^{2H} + t_2^{2H} - |t_2 - t_1|^{2H}\right).$$

- The discrete time increment process of $J(t)$

  $$\Delta J(t) = J(t+1) - J(t), \quad t = 0, 1, 2, \ldots,$$

  is stationary and LRD with long-range parameter H.

- The $2 \leq m^{\text{th}}$ order joint cumulants of the LISDLG process $J(t)$ are

  $$\mathrm{cum}(J(t_1), \ldots, J(t_m)) =$$

$$\text{const.}\sigma_0^{2m}(m-1)!\,\underset{\underline{t}}{\text{sym}}\left(\int_0^{\underline{t}} D_{\tau_0}(\underline{s})^{2(H-1)}d\underline{s}\right).$$

- The increments of the LISDLG process $J(t)$ are stationary and monofractal, i.e., $\beta(m)$ does not depend on the order of the cumulants, because

$$\text{cum}_m\left(\Delta J^{(n)}(k)\right)=k_1(m)n^{2H-2}, \qquad (4)$$

where

$$k_1(m)=(m-1)!\frac{2^{1-2H}}{1-H}c_0\sigma_0^{2m-2}\int_{[0,1]^m} D_{\tau_0}(\underline{s})^{2(H-1)}d\underline{s}. \qquad (5)$$

Note here that the cumulants above show that the process is non-Gaussian but more Gamma like.

- The spectrum of the process $\Delta J(t)$ exists and it is given by

$$S_{2,\Delta J}(\omega)= \qquad (6)$$

$$\frac{\Gamma(2H-1)}{2\pi(1-H)}\sin(H\pi)c_0\sigma_0^2\left|e^{i\omega}-1\right|^2\sum_{k=-\infty}^{\infty}\left|\omega^{(k)}\right|^{-1-2H},$$

for $\omega\in(0,2\pi)$, where $\omega^{(k)}\triangleq\omega+2k\pi$, $k\in\mathbb{Z}$, the same as the spectrum of the Discrete Fractional Gaussian Noise (DFGN). See [8] for the latter spectrum.

- The bispectrum of the process $\Delta J(t)$ exists and it is real valued and positive, namely,

$$S_{3,\Delta J}\left(\underline{\omega}_{(2)}\right)=\frac{-9ic_0\sigma_0^4}{2\pi\sin(H\pi)\Gamma(3-2H)}\prod_{j=1}^{3}(1-e^{i\omega_j}) \qquad (7)$$

$$\underset{\left(\underline{\omega}_{(3)}^{(k,\ell)}\right)}{\text{sym}}\sum_{k=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}\left[\frac{\left|\omega_1^{(k)}\right|^{2(1-H)}}{\omega_1^{(k)}\left(\omega_2^{(\ell)}\omega_3^{(k+\ell)}\right)^2}\right],$$

where $\omega_j\in(0,2\pi)$, $\omega_j^{(k)}\triangleq\omega_j+2k\pi$, $k\in\mathbb{Z}$, $j=1,2$ and $\omega_1^{(k)}+\omega_2^{(k)}+\omega_3^{(k)}=0$. The consequence of the real valued bispectrum is that the process can not be linear (see [10] for details). It means that the process is not only non-Gaussian but it is also non-linear.

The discrete time increment process $\Delta J(t)$ is applied for the modeling of different type of network traffic. If we compare our models to other fractal models (e.g. [4] or [7]) the main difference is that those models based on constructing multiplicative processes. In contrast, our model is a limit process obtained by the limit of the integrated superposition of diffusion processes with linear differencial generator. Moreover, we use cumulants instead of absolute moments because in this case the scaling properties should not change with additive constants and with summing up of independent copies of a process. The second advantage of using cumulants is that high-order frequency domain investigation can be directly applicable. Since our model is based on the increments of a continuous-state branching process with immigration and superposition of these, we can use well-known classical stochastic analysis and computation methods, which are also advantages of the model.

5 Application to Network Traffic

In this section our new monofractal model LISDLG is applied for the measured Internet traffic described in Section 2. The measured data (measured TCP data in bytes in 100ms time-unit) are considered as the increments of a LISDLG process $J(t)$, i.e. $\Delta J(t)$. The intensity diagram of the measured trace is depicted in Figure 2.

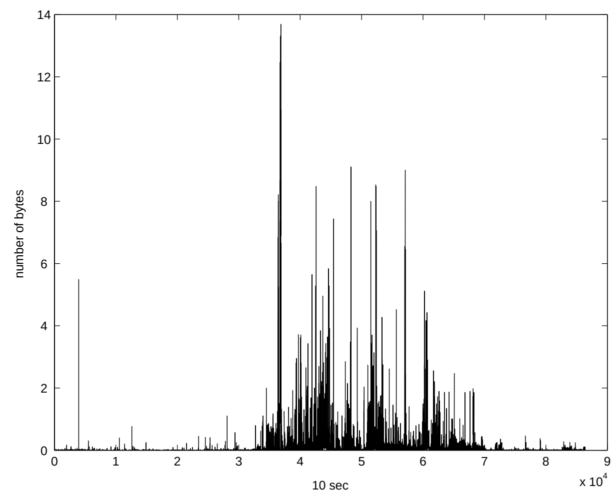

Figure 2. Intensity of the traffic

The parameters of the model are computed by fitting the empirical cumulants to the theoretical cumulants of the model. First, the empirical moments are estimated from the measured data. The empirical cumulants are calculated from the empirical moments using formula (1). For a LISDLG process by taking the logarithm in (4) we have

$$\log\left|\text{cum}_m\left(\Delta J^{(n)}(k)\right)\right|=2(H-1)\log(n)+\log k_1(m) \qquad (8)$$

For each cumulant of order m, (8) means that the dependence is linear and the lines must be parallel, each with slope $2(H-1)$. Now, for each m the common slope parameter is $2(H-1)$ and the constants are $\log k_1(m)$, $m=2,\dots,6$. Before we use equations (8) simultaneously

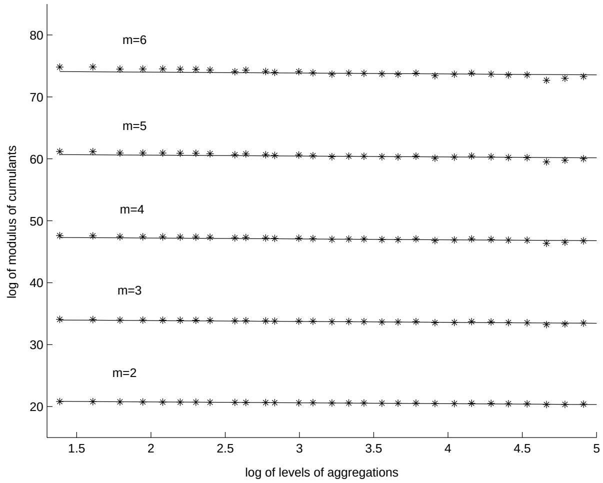

Figure 3. Theoretical (continuous lines) and estimated cumulants (discrete points)

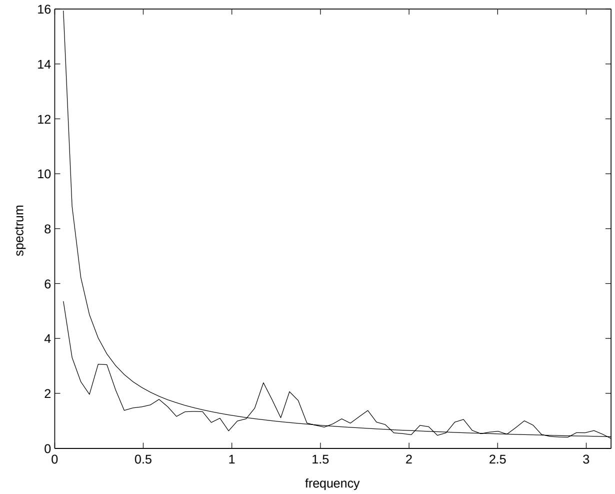

Figure 4. The estimated and the theoretical spectrum.

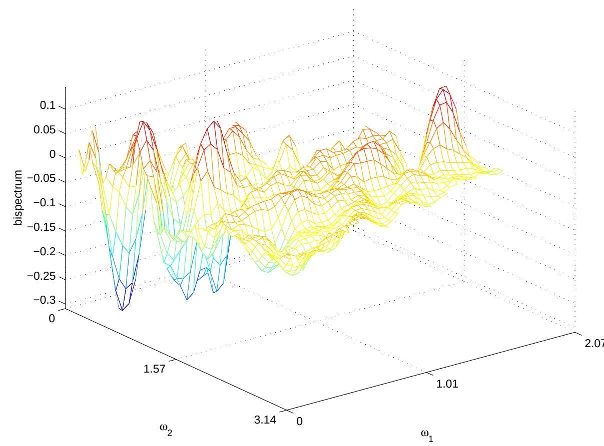

Figure 5. The imaginary part of the estimated bispectrum.

for all m fit linear regression individually when m is fixed, i.e., regress $\log\left|\operatorname{cum}_m\left(\Delta J^{(n)}(k)\right)\right|$ on $\log(n)$. The results are the estimations for $2(H-1)$, $\log k_1(m)$ and the standard deviations of the residual in the regression $v(m)$. Now the slope and the constants are estimated by weighted linear regression from the linear model with equations (8), where the weights are the inverse of the standard deviations, i.e., consider the linear model

$$\frac{\log\left|\operatorname{cum}_m\left(\Delta J^{(n)}(k)\right)\right|}{v(m)} =$$

$$2(H-1)\frac{\log(n)}{v(m)} + \frac{\log k_1(m)}{v(m)} + u,$$

for $m = 2, \ldots, 6$, where the variance of u does not depend on m. The estimated constants of formula (5) are $c_0 = 1.0445e+003$, $\sigma_0^2 = 1.9241e+005$ and $H = 0.9280$. The plot of the theoretical cumulants by (4) and the estimated one by (1) is plotted in Figure 3.

Both the estimated spectrum and the estimated bispectrum are calculated as described in Section 3.1. The theoretical spectrum and the theoretical bispectrum are computed with the estimated parameters by formula (6) and (7), respectively. The spectra are shown in Figure 4-7. Note that the imaginary part of the theoretical bispectrum is zero and therefore the imaginary part of the estimated bispectrum is fluctuating around zero with a small amplitude, see Figure 5.The real part of the theoretical bispectrum and the real part of the estimated bispectrum are plotted in Figure 6 and in Figure 7, respectively.

As the results show the model accurately captures the characteristics of the traffic in both spectrum and bispectrum domain. The well-known $1/f$ *noise phenomenon* which is a manifestation of long-range dependence in frequency domain can also be observed in both spectrum domain (around zero) and bispectrum domain (around (0,0)), see Figure 4, Figure 6 and Figure 7.

6 Conclusions

In this paper a new monofractal stochastic process called Limit of the Integrated Superposition of Diffusion processes with Linear differential Generator (LISDLG) is applied. It is demonstrated that the model can effectively characterize the monofractality of network traffic. The main

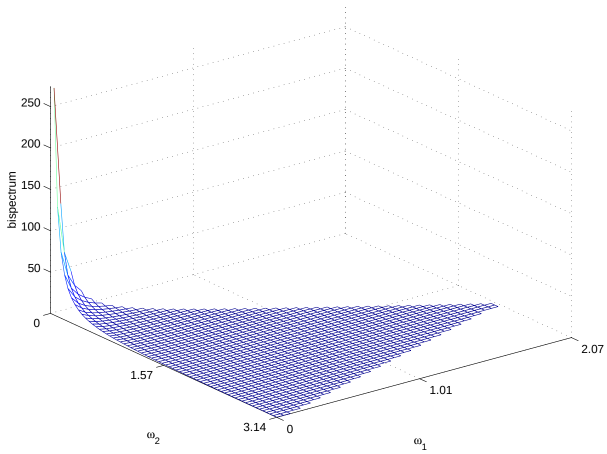

Figure 6. The real part of the theoretical bispectrum.

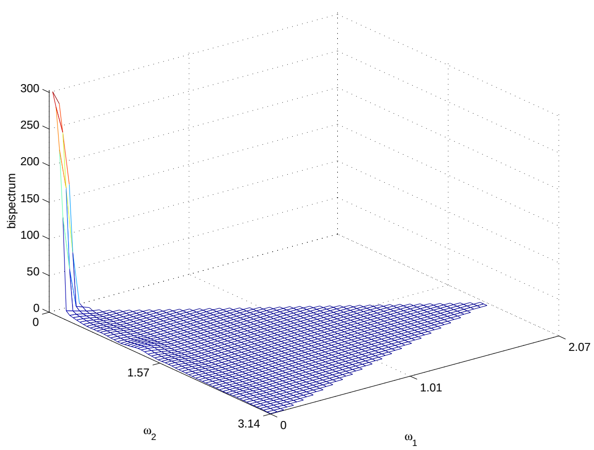

Figure 7. The real part of the estimated bispectrum.

properties of the LISDLG model are shown including covariance structure, cumulants, spectrum and bispectrum. The model captures high-order statistics by the cumulants which allow us the analysis in high-order frequency domain. The relevance and validation of the proposed model are demonstrated by an application study for measured Internet traffic.

Our future work addresses the further analysis and modeling in high-order frequency domain in order to get a deeper understanding of network traffic monofractality.

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