Capturing the Complete Multifractal Characteristics of Network Traffic

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***Abstract*-We propose a new multifractal traffic model for network traffic. The model is a combination of a multiplicative cascade with an independent lognormal process. We show that the model has all the important properties observed in data traffic including LRD, multifractality and lognormality. We also demonstrate that the model is flexible enough to capture the complete multifractal characteristics of data traffic including both the scaling function and the moment factor. On the other hand, we argue that the model is simple from practical point of view having only three parameters. Practical applications for measured data traffic and validation of the model with queueing performance evaluation are also presented.**

I. Introduction

Traffic characterization studies have shown that network traffic exhibits strong variability and burstiness on many time scales [9], [23], [4]. This was the reason to introduce fractal traffic models, which were able to capture the discovered scaling properties via *self-similarity* and *long-range dependence* (LRD) [18], [1], [23]. Self-similarity expresses the monofractal property of network traffic, that is, the traffic looks statistically similar on all (or many) time scales. LRD is revealed by the power law decay of the autocorrelation function at large lags, i.e., $r(k) \sim c_r|k|^{2H-2}, k \to \infty,\ H \in (0.5, 1)$, where c_r is a constant [1]. The degree of this slow decay is determined by the Hurst parameter (H).

In this self-similar traffic characterization framework a large number of traffic models has been developed (fractional Brownian motion (fBm) models, FARIMA models, Cox's M/G/∞ models, on/off models, etc.) [23]. From these models the fBm model [14] has widely been applied. The fBm model can nicely be handled analytically due to its Gaussian nature and it was found to be a tractable model of traffic aggregation [8].

Deeper analysis of data traffic revealed a highly irregular local structure with a more complex scaling behavior, which cannot be explained in a self-similar framework [19], [7]. It became clear that the *monofractal* traffic models (e.g. fBm) are inadequate to characterize the network traffic and *multifractals* can provide a mathematical framework for characterizing these complex local traffic structures. An exception can be found in [13], where a monofractal model is presented which has superior modelling abilities compared to self-similar models. It was also found that in many environments traffic has a non-Gaussian character, which excludes the usage of the popular Gaussian traffic models like fBm.

To develop a multifractal framework for network traffic characterization and modeling is a very recent research topic. In spite of the fact that some models are already published, the complete understanding of this phenomenon and the application of the models in practice is far from being complete.

There are different processes which are candidates for multifractal modeling. *Multiplicative cascades* were first used as a multifractal model for data traffic [15], [7]. This class is the most well-known member of the class of multifractal processes. The simplest case of this process is the *binomial cascade* which can be defined by a binary tree structure [5], [15]. Combining this process with the aforementioned fBm we can define a new class called the *fractional Brownian motions in multifractal time* [20]. This process has several nice properties, e.g. it is able to capture LRD and multifractal scaling independently. The *self-similar α-stable process* [18] is a different multifractal process. Its statistics of order $q \geq \alpha$ are not finite resulting in an irregular multifractal structure. One of the simplest process from this class is the *linear fractional stable motion*.

In this paper we propose a new multifractal model which is based on the pairwise product of a multiplicative cascade and an independent lognormal process. Earlier published multifractal models based on multiplicative cascades do not aim to capture the complete multifractal characteristics (including both the scaling function and the moment factor) and/or use many parameters for the model [16], [7], [2], [6], [10]. Comparing these models to our new model we argue that our model is simple enough for practical purposes having only three parameters but, on the other hand, flexible enough to capture accurately the whole multifractality of the traffic. We show the statistical properties of the model and also the applications for actual measured network traffic. The model is validated by comparing the queueing performance produced by the original measured traffic to the queueing results obtained by the traffic generated by our model.

The rest of the paper is organized as follows. Section II overviews the basic concepts of multifractals and multiplicative cascades. Section III presents our new multifractal traffic model including the construction of the model, its parameters and the main statistical properties. Section IV shows the application of the model for measured data traffic with its validation based on a queueing study. Finally, Section V concludes the paper and suggests some future research directions.

II. Multifractals and Multiplicative Cascades

In this section we overview the definitions of multifractals and multiplicative cascades. The iterative procedure for cascade construction is also presented.

A. Multifractals

The multifractal concept was first introduced by Mandelbrot in the context of turbulence in the early 70's. Since then multifractal processes have been widely used in a variety of research fields like geophysics, image processing, stock market modeling, and recently network traffic characterization.

A stochastic process $X(t)$ is called multifractal if it has stationary increments and satisfies [5]

$$\mathbb{E}(|X(t)|^q) = c(q)t^{\tau(q)+1} = c(q)t^{\tau_0(q)}, \qquad (1)$$

for some positive values $q \in Q$, $[0,1] \subset Q$, where $\tau(q)$ is called the *scaling function* and the *moment factor* $c(q)$ is independent of t. In this paper we refer to $\tau_0(q)$ as the scaling function instead of $\tau(q)$. An easy consequence of the definition is that $\tau(q)$ is a concave function [5]. If $\tau(q)$ is linear in q the process is called monofractal, otherwise it is multifractal. It can be shown that in the special case of self-similar process with index H we get $\tau(q) = qH - 1$ and $c(q) = \mathbb{E}(|X(1)|^q)$.

The definition above describes multifractality in terms of process moments and it may lead to a more intuitive understanding of multifractality. However, there is an alternative approach to multifractals, also found in the literature, which is based on the study of the local erratic behaviour of the process by means of its local Hölder exponents. For the details of this approach, see [15] and references therein. The most obvious examples of multifractals are self-similar and multiplicative processes.

B. *Multiplicative cascades*

The simplest multifractals are typically constructed by an iterative procedure called multiplicative cascade. Consider a unit interval associated with a unit mass. At state $k = 1$ divide the unit interval into two equal subintervals and associate with them the mass r and $1 - r$, respectively. The fraction r is called the multiplier. The same rule is applied to each subinterval and its associated mass. The multipliers r are chosen to be independent random variables R concentrated on $[0,1]$ with the probability distribution function $F_R(x)$, $\mathbb{E}(R) = 1/2$. We also choose the multiplier r to have a symmetric density function so that r and $1 - r$ have the same marginal distribution. Thus at the state k a dyadic interval of length $\triangle t_k = 2^{-k}$ starting at $t = 0.\eta_1...\eta_k = \sum \eta_i 2^{-i}$ has the mass (measure)

$$\mu(\triangle t_k) = R(\eta_1)R(\eta_1,\eta_2)\dots R(\eta_1,\dots,\eta_k),$$

where $R(\eta_1,\dots,\eta_i)$ indicates the multiplier at state number i. Since multipliers are i.i.d. it is easy to show that the measure μ satisfies the scaling relationship:

$$\mathbb{E}(\mu(\triangle t_k)^q) = (\mathbb{E}(R^q))^k = \triangle t_k^{-\log_2 \mathbb{E}(R^q)},$$

which defines a multifractal process with scaling function $\tau_0(q) = -\log_2 \mathbb{E}(R^q)$.

Note that the multifractal process constructed above is also referred to *conservative cascade*. An important property of this random cascade is its dependence structure due to the construction. If the multipliers used in the construction have the same fixed value r_0 ($0 < r_0 < 1$) then the obtained multiplicative measure is called *binomial*. Binomial measure is a deterministic cascade, its scaling function being $\tau_0(q) = -\log_2(r_0^q + (1-r_0)^q) + 1$. In addition, if the iteration only conserves mass on the average,i.e., multipliers at each mass division are also i.i.d. but have mean of 1/2, the corresponding measure is called *canonical* [5].

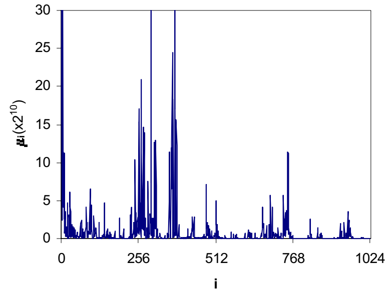


Fig. 1. Conservative cascade measures ($R \sim$ Uniform$(0,1)$) at stage $k = 10$.

An example of the multiplicative cascades can be seen in Fig. 1.

From the network modeling point of view only conservative cascades are of interest. The binomial cascade is naturally excluded because it is a deterministic process. The canonical cascade cannot be used since it is an independent random process, while network traffic flows are long-range dependent. In this study we use the conservative cascade as a building block of our traffic model.

III. A Multifractal Traffic Model

Our multifractal model based on multiplicative cascades is presented in details in this section. Statistical properties of the model are also derived. Finally, we give a comparison to the different multifractal models which can be found in teletraffic literature.

A. *Construction of the model*

Suppose that the multifractal analysis of a real data series obtained form measured network traffic shows its multifractal properties characterized by the scaling function $\tau_0(q)$ and the moment factor $c(q)$. The obvious task of cascade modeling is to find a convenient probability distribution for multipliers R such that $-\log_2(\mathbb{E}(R^q)) = \tau_0(q)$. However, this cascade model captures only the multifractal properties given by the scaling function and fails to furnish any information on the moment factor $c(q)$. The idea for a more comprehensive traffic model is the following: 2^N synthetic data $\mu(\triangle t_N)$ is first generated by multiplicative cascade with multipliers governed by the distribution of R. Then 2^N data series of our model is the pairwise product of the cascade data series and an i.i.d. random samples of a positive random variable Y with the same length. The variable Y is chosen to be independent of the cascade measure $\mu(\triangle t_N)$, thus the obtained series, denoted by $X(\triangle t_N)$, satisfies

$$\mathbb{E}(X(\triangle t_N)^q) = \mathbb{E}(Y^q)\mathbb{E}(\mu(\triangle t_N)^q) = \mathbb{E}(Y^q)\triangle t_N^{\tau_0(q)}. \qquad (2)$$

The model fitting task is to find the suitable random variables R and Y such that

$$\begin{cases} -\log_2(\mathbb{E}(R^q)) & = & \tau_0(q) \\ \mathbb{E}(Y^q) & = & c(q). \end{cases}$$

The presented model is relevant to multifractal network traffic for the following reasons. First, it is based on the multiplicative construction of a cascade which seems to closely match the TCP/IP protocol operating mechanics as suggested in a number of traffic research studies [3], [7] as the main cause of multifractality in traffic data at small timescales. Second, the model traffic can be interpreted as the product of the random peak rate of the flow Y and the measure of burstiness $\mu(\triangle t_N)$ at the modelled time scale $\triangle t_N$.

For practical use, we introduce some modifications to the model. The measure $\mu(\triangle t_N)$ has a very small value since it is the product of N multipliers $0 < r < 1$, so to avoid loss of information we multiply the cascade measures by 2^N. Since $\mathbb{E}(\mu(\triangle t_N)) = 2^{-N}$, this normalizes the cascade increment so that it has unit mean. As another modification, we also rescale the cascade process to have unit time interval at stage N instead of $\triangle t_N = 2^{-N}$. For a multifractal increment $X_\triangle$

$$\begin{aligned}\mathbb{E}(X_\triangle^q) &= c_1(q)\triangle t_1^{\tau_0(q)}\\ &= c_1(q)\left(\frac{\triangle t_2}{\triangle t_1}\right)^{-\tau_0(q)}\triangle t_2^{\tau_0(q)} = c_2(q)\triangle t_2^{\tau_0(q)}, \quad (3)\end{aligned}$$

which means that the choice of time unit influences the value of the moment factor $c(q)$. After applying these changes to the model we obtain

$$\mathbb{E}(X(\triangle t_0)^q) = \mathbb{E}(Y^q)2^{N[q+\log_2 \mathbb{E}(R^q)]}\triangle t_0^{-\log_2 \mathbb{E}(R^q)}, \quad (4)$$

where $\triangle t_0$ denotes the unit time interval of the data traffic to be modelled.

B. Model parameters

For multifractal traffic data, the scaling function $\tau_0(q)$ and the logarithm the moment factor $c(q)$ can be estimated by a simple absolute moment method, see [12] for details. Denote these estimated functions by $\tilde{\tau}_0(q)$ and $\log \tilde{c}(q)$, respectively. Owing the modifications mentioned above, the random variable R and Y should be chosen such that

$$\begin{aligned}-\log_2(\mathbb{E}(R^q)) &= \tilde{\tau}_0(q) \quad (5)\\ \log \mathbb{E}(Y^q) &= \log \tilde{c}(q) - [q + \log_2 \mathbb{E}(R^q)]N\log 2\\ &= \log \tilde{c}(q) - [q - \tilde{\tau}_0(q)]N\log 2. \quad (6)\end{aligned}$$

Our analysis of various measured traffic with multifractal properties shows that the choice of R as a *symmetric beta random variable* on [0,1] Beta(α,α) with only one parameter $\alpha > 0$ is accurate to model the estimated scaling function. In this case

$$\tau_0(q) = \log_2 \frac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)}, \quad (7)$$

where $\Gamma(.)$ denotes the Gamma function.

We also choose the *lognormal distribution* for the random variable Y. It has two parameters m and σ and the moment is of the form $\mathbb{E}(Y^q) = e^{mq+\sigma^2q^2/2}$. Thus from Eq. (6) m and σ should be chosen such that

$$mq + \frac{\sigma^2q^2}{2} = \log \tilde{c}(q) - \left\{q - \log_2 \frac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)}\right\} N\log 2. \quad (8)$$

In summary, our presented multifractal model has three parameters (α, m, σ) and the following characterization functions:

$$\left\{\begin{aligned}\tau_0(q) &= \log_2 \tfrac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)}\\ c(q) &= e^{mq+\sigma^2q^2/2}2^{N\left(q-\log_2 \frac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)}\right)}.\end{aligned}\right.$$

Note that if the right-hand side of Eq. (6) is a concave function of q then the examined traffic data cannot be captured by our model since the absolute moment of any stochastic process is a *log-convex* function of the moment order q. (This property is easily derived from the Hölder inequality.) These kind of multifractal traffic cannot be fully characterized by a cascade based model.

C. Statistical properties

Statistical properties of multiplicative cascades are studied in a number of papers, see [22], [6] for examples. We extend these properties for 2^N synthetic samples of our multifractal model.

(i) As an extension of the conservative cascade our traffic model is an exact *positive multifractal process*. In the model construction presented in the previous section, the multifractality is characterized by scaling function $\tau_0(q) = \log_2 \frac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)}$ and the logarithm of the moment factor $\log c(q) = (mq + \sigma^2q^2/2) + \left(q - \log_2 \frac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)}\right) N\log 2$ where (α, m, σ) are model parameters.

(ii) The mean and the variance of the model process are:
$\mathbb{E}[X(\triangle t_0)] = \mathbb{E}(Y) = e^{m+\sigma^2/2}$;
var$[X(\triangle t_0)] = \mathbb{E}(Y^2)2^{2N}\mathbb{E}(R^2)^N - \mathbb{E}[X(\triangle t_0)]^2 =$
$= e^{2m+2\sigma^2}(\frac{\alpha+1}{\alpha+1/2})^N - e^{2m+\sigma^2}$.

(iii) For $N \gg 1$, $X(\triangle t_0)$ has lognormal distribution. This property is deduced directly from the fact that $X(\triangle t_0) = 2^N.Y.R(\eta_1)\ldots R(\eta_1,\ldots,\eta_N)$ and the central limit theorem.

(iv) $X(\triangle t_0)$ has long-range dependent correlation structure. Consider the covariance cov$[X(\triangle t_0)_n, X(\triangle t_0)_{n+k}]$, where $k = 2^p, p = 1, 2, \ldots$, which can be derived as follows:

$$\begin{aligned}&\text{cov}[X(\triangle t_0)_n, X(\triangle t_0)_{n+k}] =\\ &= \mathbb{E}(Y)^2\left\{2^{2N}\mathbb{E}[\mu(\triangle t_N)_n.\mu(\triangle t_N)_{n+k}] - 1\right\}. \quad (9)\end{aligned}$$

The two measures $\mu(\triangle t_N)_n$ and $\mu(\triangle t_N)_{n+k}$ have the same origin at stage $N-p-1$, denoted by $\mu(\triangle t_{N-p-1})$, thus $\mu(\triangle t_N)_n = \mu(\triangle t_N)_{N-p-1}.r_{N-p}\prod_{i=N-p+1}^N r_{i,j_1}$ and $\mu(\triangle t_N)_{n+k} = \mu(\triangle t_N)_{N-p-1}.(1-r_{N-p})\prod_{i=N-p+1}^N r_{i,j_2}$ where $r_{i,j}$ denotes the actual multiplier values at stage i. Then $\mathbb{E}[\mu(\triangle t_N)_n.\mu(\triangle t_N)_{n+k}] = \mathbb{E}[\mu(\triangle t_{N-p-1})^2]\mathbb{E}[r_{N-p}(1-r_{N-p})]\mathbb{E}[\prod_{i=N-p+1}^N r_{i,j_1}r_{i,j_2}] = \mathbb{E}(R^2)^{N-p-1}[\frac{1}{2}-\mathbb{E}(R^2)](1/2)^{2p}$. Insert this into Eq. (9) we get

$$\begin{aligned}&\text{cov}[X(\triangle t_0)_n, X(\triangle t_0)_{n+k}] =\\ &= e^{2m+\sigma^2}\left\{\frac{\alpha(\alpha+1)^{N-1}}{(\alpha+1/2)^N}\left[\frac{\alpha+1}{\alpha+1/2}\right]^{-p} - 1\right\}\\ &= e^{2m+\sigma^2}\frac{\alpha(\alpha+1)^{N-1}}{(\alpha+1/2)^N}k^{-\log_2\left(\frac{\alpha+1}{\alpha+1/2}\right)} - e^{2m+\sigma^2}. \quad (10)\end{aligned}$$

Thus when N, k are large the covariance is ruled by $k^{-\log_2\left(\frac{\alpha+1}{\alpha+1/2}\right)}$, i.e., the model has LRD structure with Hurst parameter $H = 1 - \frac{\log_2 \frac{\alpha+1}{\alpha+1/2}}{2}$. It is easy to check that for $\alpha > 0$ $H \in (0.5, 1)$.

D. Comparison with other multifractal models

We first summarize the reasons which explain the suitability of the described multifractal model for network traffic modeling: (1) it is a positive process, hence reasonable for the simulation of the traffic counting processes; (2) it captures the full multifractal characteristics defined by the scaling function $\tau_0(q)$ and the moment factor $c(q)$; (3) it has approximately lognormal marginal distribution, which seems to match the real traffic; (4) it also has LRD correlation structure, which is an important property of high-speed LAN/WAN network traffic.

Since the observation of the flexible scaling structure in some WAN traffic environments [17], [11], [3] network researchers have suggested several multifractal models for characterization of these traffic flows among which two distinct approaches can be identified. The first one uses multifractal time for subordinating a monofractal process (FBM) to model multifractals, e.g. in [21]. The disadvantages of this approach lie in the presence of some negative values and Gaussian marginals of the model synthetic processes, which is not suitable for network traffic simulation. The other approach is based on the multiplicative cascades [16], [7], [2], [6], [10]. In general, multiplicative cascades are very attractive for traffic modeling. They are positive processes, easy to generate, and also possessing a plausible explanation for the origin of multiscaling properties in the traffic [7]. However, authors in [2] and [6] only fit the model to the LRD structure of the measured traffic, thus these models do not capture the multifractal characteristics of the traffic which is also ruled by the higher order moments. Cascade models in [16], [7] provide a better fit to the traffic multifractality but require N parameters for 2^N synthetic data. Then it is difficult to use these models in analytical approaches, e.g., in queueing performance estimations for multifractal traffic input.

In contrast, out traffic model suggests an alternative method of cascade modeling with only three parameters. The model provides the closed analytic form of the characteristic functions of multifractality with properties close to the real data traffic.

IV. Analysis

In this section we examine the effectiveness of the model in simulation of some real data traces. After fitting the parameters, synthetic data is generated and compared with the real traces in a queueing performance analysis.

A. Model fitting

Data traces were collected from our real traffic measurements carried out at an outgoing Internet connection of the Informatics Building, Budapest University of Technology and Economics in 2000. The traffic traces, captured by `tcp-dump`, were the aggregated traffic of about 100 workstations used by staff member, PhD students, and student laboratories. We use two data sets, denoted by DATASET1 and DATASET2, for analysis. Both sets contain 2^{17} data samples of IP traffic bytes, counted in 100 byte units for simple calculation, arriving in consecutive time intervals of 60ms. The absolute moment based method presented in [12] was used to test the multifractal characteristics of the traces.

Figure 2 presents the multifractal analysis and model fitting results for the DATASET1. As seen in Fig. 2(a) the concavity of the estimated scaling function shows evidence for multiscaling structure of the DATASET1. We can also see in this figure the theoretical multiscaling function of the multiplicative cascade with parameter $\alpha_1 = 9.36$ of the Beta(α, α)-distributed multipliers. As it is seen, the multiscale function of the cas-

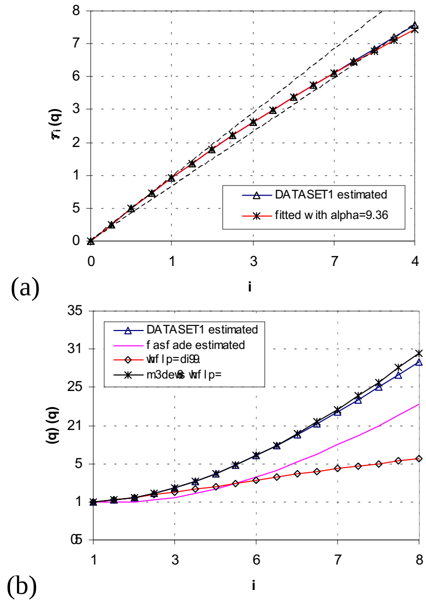


Fig. 2. Model fitting results for the DASTASET1: (a) the estimated scaling function and the fitted curve; (b) moment factor fitting.

cade process (also of the model) provide a very tight fit to the estimated curve. The next step is to determine the value of the parameters m and σ of the lognormal distribution to account for the difference between the logarithm of the estimated moment factor and the moment factor of the multiplicative cascade. The two moment factors and their difference are plotted in Fig. 2(b). We find that the lognormal distribution with $m_1 = 0.57$ and $\sigma_1 = 0.23$ is adequate for this goal.

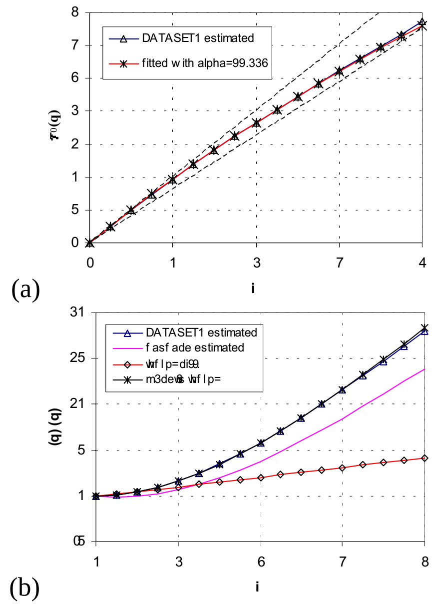


Fig. 3. Model fitting results for the DASTASET2: (a) the estimated scaling function and the fitted curve; (b) moment factor fitting.

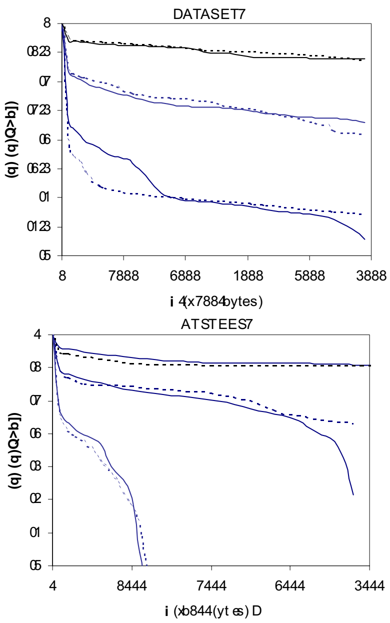
A similar procedure is carried out for the DATASET2 and the results are presented in Fig. 3. The analysis also indicates the multiscaling properties for this data set. Our model can be fitted to the estimated multifractal characteristic functions with

$\alpha_2 = 11.003$, $m_2 = 0.46$, and $\sigma_2 = 0.15$.

B. Queueing analysis

Next we study the queueing performance of the real data traces and their simulated data sets using the new model. We consider an infinite-buffer single-server queue with constant service rate and FIFO serving discipline. The synthetic data sets for each real trace are generated using the estimated model parameters. The analysis results are shown in Fig. 4.

We present in Fig. 4(a) the observed queue tail probabilities for the DATASET1 and their synthetic sets. The analysis is considered at different server utilizations ρ. In the plot the solid lines present the tail probabilities of the real trace and the dashed lines the results of the trace simulation. The curves, from top to bottom, correspond to $\rho = 0.9$, 0.7, and 0.5, respectively. The results show that the queueing behaviour of the real trace and their synthetic sets are approximately similar.



(a)

(b)

Fig. 4. Experimental queue tail distributions of the DATASET1 and DATASET2 (solid lines) and their corresponding synthetics (dashed lines) at different server utilizations of 0.9, 0.7, and 0.5 (from top to bottom).

Queueing performance comparison on the DATASET2 and its synthetics, as seen in Fig. 4(b), also exhibits a very good match. The results show that our multifractal traffic model provides queueing behaviour close to that of the measured traffic.

V. CONCLUSION

A new multifractal traffic model has been introduced in this paper. The modelling process is the pairwise product of a multiplicative cascade and an independent, identically distributed lognormal process. The obtained traffic model thus can capture the full characteristics of multifractality defined by its scaling function and the moment factor. We also study the detailed statistical properties of the model and find that it can match the most important properties of the real WAN traffic like long-range dependence and lognormal marginals.

The traffic model is then applied to two real traffic traces which are found to have multiscaling structure. The real traces and their corresponding synthetic sets generated by the model are compared in a queueing performance test of a infinite-buffer single-server queueing system with constant service rate. The results show that the queue tail behaviours, simulated at different server utilizations, are very close to each other. We conclude that our model provides a good alternative method for multifractal traffic modeling.

The model construction possesses the opportunities for further developments. One can choose other distributions which provide an even better fit for multifractal characteristic functions. We also intend to apply the model to more measured data traffic with multifractal properties.

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