May 25-26, 1994 COST 242 TD(94)014

THE EFFECT OF MULTIPLEXING CDV AFFECTED CBR CELL STREAMS

Sándor Molnár
Technical University of Budapest
Department of Telecommunications and Telematics
Sztoczek u. 2
H-1111 Budapest, Hungary

Søren Blaabjerg
Ellemtel
Scheelevägen 17
S-223 70 Lund
Sweden

Abstract: This study deals with the analysis of delay and loss performance of an ATM multiplexer receiving Constant Bit Rate (CBR) cell streams which have been exposed to Cell Delay Variation (CDV). The CBR cell stream are exposed to CDV in a single FIFO multiplex described by a diffusion model. Furthermore, the queueing performance of the superposition of such cell streams when offered to a FIFO queue is carried out by an application of the Benes result.
The study also considers the CDV behaviour after the CBR cell stream has passed through a series of queues based on a simple model. All investigations are illustrated by numerical examples.

1 Introduction

To understand the alteration of cell stream characteristics in ATM networks is and important issue with implications on traffic and congestion control. As the cell stream passes through the network its characteristics is changing due to the interference with other cell streams. Each cell will experience different traffic conditions in buffers, and therefore different delays resulting in a jittered cell stream. This phenomenon, denoted as Cell Delay Variation (CDV), is a major issue of the standardization bodies [CCITT] as well as ATM forum, and also in the literature many contributions can be found [Gro91], [Bla93], [Hue94].

So far, most of the efforts have been made in describing a single CDV affected cell stream [Bla93], [BMS93], in dimensioning the UPC-algorithm (leaky bucket) [Hue94], [GBDR92], [CTF92], and deriving the effect CDV has on peak rate enforcement [GRo91], [Skl94].

The objective of this paper is to present an analysis of the delay and loss performance of an ATM multiplexer receiving a number of CDV affected CBR cell streams. To get more insight into the effect of CDV after a series of queues is another objective.

In section 2 a brief description of a diffusion model can be found, by which we model the CDV affected CBR cell stream. The probability distributions for the number of arrivals in both an arbitrary window and in a window starting from an arrival are presented. In section 3 we investigate the superposition of these cell streams and use the "Benes result" to obtain an accurate approximation for the queue length distribution. The approach is illustrated by numerical examples. In section 4 a study about the effect CDV created from a series of queues is carried out based on [BRe92] with numerical examples.

2 Characteristics of a CBR cell stream after passage of an ATM Multiplex

In order to characterize the effect CDV has when a CBR stream is multiplexed with a background stream, we restrict our attention to a FIFO queue with deterministic service time receiving information from two cell arrival processes, a CBR cell arrival process with constant inter arrival time d, and a background cell arrival process with arrival intensity λ. The time unit is chosen as the service time of one cell, and the load on the FIFO queue is $\rho = \lambda + 1/d$.

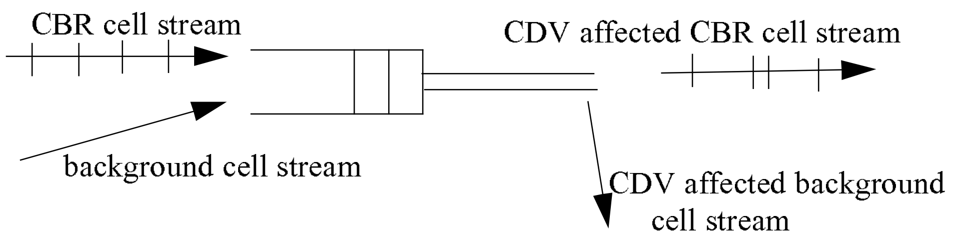

Figure 1 The FIFO model and the CDV affected cell stream

Let d denote the interarrival time between CBR cells at the source and choose the time such that cell no. n is transmitted at time nd from the source.

Let W_n denote the delay of cell no. n. The sequence W_n is assumed stationary and the dependence between successive CBR cell delays is assumed Markovian.

Define $\tau_n = nd + W_n$. Thus τ_n denotes the departure time of cell no. n (See Figure 2). Also define the inter exit time of cell no. n as: $U_n = \tau_n - nd - \tau_0$

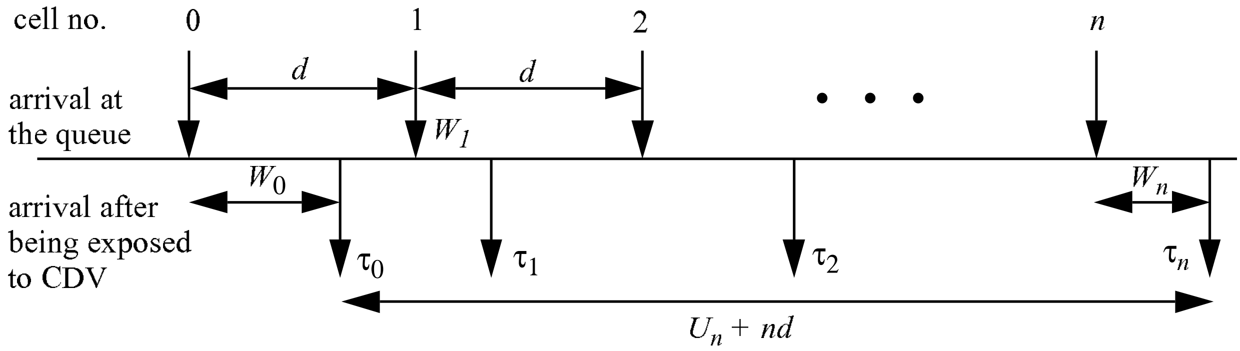

Figure 2 The inter exit time distribution is the difference between the actual arrival of cell no. n and the expected arrival time.

In section 2.1 we consider the case in which the load is smaller than 1 and apply a diffusion analysis to characterize the CDV affected CBR stream after passage of the queue. In section 2.2 we assume that the load of the multiplex is 1, and show that in many cases the CDV affected stream is well approximated by a renewal process and we find the distribution of the interarrival time.

2.1 The Diffusion Approximation

In order for the diffusion approach to work we need to assume that the deterministic server queue receiving the superposition of the CBR stream and the background stream obeys an exponential asymptotic behaviour i.e.

$$P\{W > r\} \approx C\exp(-zr) \qquad \text{for} \quad r \to \infty \tag{1}$$

for some positive real number z, where W denotes the virtual waiting time.

2.1.1 Virtual Waiting Time Behaviour Described by Diffusion

We assume that the dependencies between successive CBR cell delays in the queue are markovian as in [GRo91], but in order to obtain closed form expressions, the evolution of the queue length (or virtual waiting time) between CBR arrivals will be modelled by a reflected Brownian motion.

Let $\tilde{W}_t$ denote the waiting time a fictitious observer would experience if he joined the diffusion queue at time t (the virtual waiting time at time t). The probability of $\tilde{W}_t \le x$ conditioned on $\tilde{W}_0 = y$ is, for a Brownian motion with drift m (assumed smaller than zero in order to ensure a stable queue), variance σ^2 and a reflection in zero, in section 2.8 in [Kle76] derived to be:

$$P\{\tilde{W}_t \le x | \tilde{W}_0 = y\} = \begin{cases} \Phi\left(\dfrac{x-y-mt}{\sigma\sqrt{t}}\right) - e^{2\frac{m}{\sigma^2}x}\,\Phi\left(\dfrac{-x-y-mt}{\sigma\sqrt{t}}\right) & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases} \tag{2}$$

where Φ denotes the standard Gaussian probability distribution.

The right hand side of (2) is a distribution function in x for all $y \ge 0$ and all $t > 0$, and it converges towards the exponential distribution with mean $-\sigma^2/2m$, independent of initial condition y, when t tends to infinity.

Both the arrivals of the CBR cells as well as the cells from the background process are modelled in accordance with the diffusion approach, i.e. the queue length is not increased by one at each CBR cell arrival. Instead these arrivals as well as background arrivals and departures are taken into account by a proper choice of drift as well as variance in the diffusion process as shown next.

The delay of CBR cell no. n, W_n is approximated by:

$$W_n = \tilde{W}_{nd} \tag{3}$$

that is *the waiting time that CBR cell no. n experience in the FIFO multiplex is approximated by the virtual waiting time in the diffusion queue at time nd.*

A disadvantage with formula (2) is that there is a positive probability that the waiting time between time t and time $t + u$ decreases more than u which of course in the original M+D/D/1 queue is impossible. If t is the time of CBR arrival no. n and $u = d$ then this would imply that CBR cell no. n+1 would depart from the queue before cell no. n. This weakness is inherent to the model and therefore the model should only be used for values of d for which this probability is low, implying that d needs to be large.

When the background traffic is Poissonian, the asymptotic behaviour for large queue length have been worked out in [BSi87]. They found that the number of cells in the queue just prior to a CBR arrival have a geometric tail determined by the root z_∞ of $z^{d-1} = e^{\left(\rho - \frac{1}{d}\right)d(z-1)}$ outside the unit disk with smallest module. In our case with positive probability of a single arrival, [BSi87] shows that this root is unique, real and greater than one. The asymptotic slope of the waiting time therefore should be exponential with slope $s_\infty = ln(z_\infty)$. By choosing the variance parameter σ^2 in the diffusion process properly, we are able to match the decay rates of the stationary waiting time distribution in the diffusion approximation with the correct one of the M+D/D/1 queue.

Since the diffusion decay rate is $-2m/\sigma^2$ we get:

$$\sigma^2 = -\frac{2m}{ln(z_\infty)} \qquad \text{and} \qquad m = \rho - 1 \tag{4}$$

where we as drift m have taken the same as in the M/D/1 queue.

If the background traffic is described by the Discrete time Markovian Arrival Process (DMAP) the superposition with the CBR source is also an DMAP and since the asymptotic decay rate of the DMAP/D/1 queue is found by computing the dominant root of a specific determinant function which involves the Perron Frobenious eigenvalue, see [GCa92] for details, the diffusion approximation is also applicable in this case only with the variance parameter changed.

2.1.2 The Probability Distribution of the Inter Exit Time and the Interarrival Time

The inter exit time in the diffusion context is: $\tilde{U}_t = \tilde{W}_t - \tilde{W}_0 - t$ (think of $t = nd$), and as shown in [Bla93] it is given as

$$\tilde{F}_t(x) = P\{\tilde{U}_t \le x\} = \begin{cases} \frac{1}{2} + \frac{1}{2}\Phi\left(\frac{x - mt}{\sigma\sqrt{t}}\right) - \frac{1}{2}e^{\frac{2m}{\sigma^2}x}\Phi\left(-\frac{x + mt}{\sigma\sqrt{t}}\right) & \text{for } x > 0 \\ \frac{1}{2}e^{-\frac{2m}{\sigma^2}x}\Phi\left(\frac{x - mt}{\sigma\sqrt{t}}\right) + \frac{1}{2}\Phi\left(\frac{x + mt}{\sigma\sqrt{t}}\right) & \text{for } x < 0 \end{cases} \tag{5}$$

The interarrival time distribution between cell no. k and $k+n$ is trivially obtained from the inter exit time distribution as

$$F_n(x) = P\{\tau_{k+n} - \tau_k \le x\} = P\{\tau_n - \tau_0 \le x\} = P\{\tilde{U}_{nd} \le x - nd\} \tag{6}$$

which is simply the inter exit distribution shifted by nd.

2.1.3 The Number of Arrivals in a Window Starting Just after an Arrival

Consider an interval of the form $]\tau_j, \tau_j + t]$ starting just after the arrival of cell no. j. Then

$$P\{N(\tau_j, \tau_j + t) \ge n\} = P\{\tilde{U}_{nd} + nd \le t\} = \tilde{F}_{nd}(t - nd) =$$

$$\begin{cases} \frac{1}{2} + \frac{1}{2}\Phi\left(\frac{t}{\sigma\sqrt{nd}} - \frac{(1+m)}{\sigma}\sqrt{nd}\right) - \frac{1}{2}e^{\frac{2m}{\sigma^2}(t - nd)}\Phi\left(-\frac{t}{\sigma\sqrt{nd}} + \frac{(1-m)}{\sigma}\sqrt{nd}\right) & \text{for } t > nd \\ \frac{1}{2}e^{-\frac{2m}{\sigma^2}(t - nd)}\Phi\left(\frac{t}{\sigma\sqrt{nd}} - \frac{(1+m)}{\sigma}\sqrt{nd}\right) + \frac{1}{2}\Phi\left(\frac{t}{\sigma\sqrt{nd}} - \frac{(1-m)}{\sigma}\sqrt{nd}\right) & \text{for } t < nd \end{cases} \tag{7}$$

by (5) where $N(\tau_j, \tau_j + t)$ denotes the number of arrivals in $]\tau_j, \tau_j + t]$.

2.1.4 The Number of Arrivals in an Arbitrary Window

Let t_0 denote an arbitrary point in time, and consider the interval $]t_0, t_0+t]$. According to section 4.2 in [CLe66] then

$$P\{N(t_0, t_0+t) \geq n\} = P\{Y + X_2 + .. + X_n \leq t\} = \frac{1}{d}\int_0^t (F_{n-1}(u) - F_n(u))\,du \qquad (8)$$

in which Y denote the forward recurrence time, X_i denote the interarrival time between cell no. i-1 and i, $F_i(u)$ denotes the distribution function of $\sum_{j=1}^{i} X_j$, and $N(t_0, t_0+t)$ denotes the number of arrivals in $]t_0, t_0+t]$. (8) is a direct consequence of formula (4.2.10) in [CLe66]. Since $F_n(u) = P\{\tilde{U}_{nd} \leq u - nd\} = \tilde{F}_{nd}(u - nd)$ is given in closed form in (7) we may obtain the probability distribution of the number of arrivals in an arbitrary window by integrating (7) with respect to t. The result can be formulated as:

$$P\{N(t_0, t_0+t) \geq n\} = G_{n-1}(t) - G_n(t) \qquad (9)$$

in which $G_n(t)$ for $t < nd$ is given as

$$\begin{aligned}
G_n(t) = {} & \frac{\sigma^2}{(-4m)}\frac{1}{d}\Big\{ e^{-\frac{2m}{\sigma^2}(t-nd)}\Phi\Big(\frac{t}{\sigma\sqrt{nd}} - \frac{1+m}{\sigma}\sqrt{nd}\Big) - e^{\frac{2m}{\sigma^2}nd}\Phi\Big(-\frac{1+m}{\sigma}\sqrt{nd}\Big) \\
& - \Phi\Big(\frac{t}{\sigma\sqrt{nd}} - \frac{1-m}{\sigma}\sqrt{nd}\Big) + \Phi\Big(-\frac{1-m}{\sigma}\sqrt{nd}\Big)\Big\} \\
& + \frac{1}{2d}\Big\{ (t-(1-m)nd)\,\Phi\Big(\frac{t}{\sigma\sqrt{nd}} - \frac{1-m}{\sigma}\sqrt{nd}\Big) + (1-m)nd\,\Phi\Big(-\frac{1-m}{\sigma}\sqrt{nd}\Big) \\
& + \sigma\sqrt{nd}\,\varphi\Big(\frac{t}{\sigma\sqrt{nd}} - \frac{1-m}{\sigma}\sqrt{nd}\Big) - \sigma\sqrt{nd}\,\varphi\Big(-\frac{1-m}{\sigma}\sqrt{nd}\Big)\Big\}
\end{aligned} \qquad (10.a)$$

and for $t > nd$, $G_n(t)$ is given as:

$$
\begin{aligned}
G_n(t) \;=\; & \frac{\sigma^2}{(-4m)}\frac{1}{d}\Big\{ e^{\frac{2m}{\sigma^2}(t-nd)} \Phi\Big(\frac{-t}{\sigma\sqrt{nd}} + \frac{1-m}{\sigma}\sqrt{nd}\Big) - e^{\frac{2m}{\sigma^2}nd}\Phi\Big(-\frac{1+m}{\sigma}\sqrt{nd}\Big) \\
& - \Phi\Big(\frac{-t}{\sigma\sqrt{nd}} + \frac{1+m}{\sigma}\sqrt{nd}\Big) + \Phi\Big(-\frac{1-m}{\sigma}\sqrt{nd}\Big)\Big\} \\
& + \frac{1}{2d}\Big\{ ((1+m)nd - t)\Phi\Big(\frac{t}{\sigma\sqrt{nd}} - \frac{1-m}{\sigma}\sqrt{nd}\Big) + (1-m)nd\Phi\Big(-\frac{1-m}{\sigma}\sqrt{nd}\Big) \\
& + \sigma\sqrt{nd}\varphi\Big(\frac{-t}{\sigma\sqrt{nd}} + \frac{1+m}{\sigma}\sqrt{nd}\Big) - \sigma\sqrt{nd}\varphi\Big(-\frac{1-m}{\sigma}\sqrt{nd}\Big)\Big\} \\
& + \frac{t-nd}{d}
\end{aligned}
\tag{10.b}
$$

where we have utilized the symmetry relation $\tilde{F}_t(x) + \tilde{F}_t(-x) = 1$ to obtain (10.b).

2.2 When the Load on the Multiplex is One

Now, consider the case that the load of the multiplex is exactly 1. This case has been considered by both [BMS93] and [Kel93]. Here we shall apply the arguments from [Kel93]. The probability that the queue is empty can be neglected, and we get that the difference in waiting time between CBR cell no. i-1 and CBR cell no i is

$$
W_i - W_{i-1} = 1 + N_{\text{background}}(d(i-1), di) - d \tag{11}
$$

where $N_{\text{background}}(s, t)$ denotes the number of arriving cells from the background stream in the time interval $]s,t]$

If the background stream constitutes a Poisson process, then $N_{\text{background}}(s, t)$ is a Poisson distribution with parameter $\lambda(t-s)$. From (11) we therefore derive the result that *the CDV affected CBR stream constitutes a renewal process with interarrival time distribution* $1 + P(\lambda d) = 1 + P(d-1)$ where P denotes the Poisson distribution. The independence between different interarrival times comes from the memoryless property of the exponential distribution in the Poisson process. The result implies that the squared coefficient of variation of the CDV affected stream is $(d\text{-}1)/d^2$.

If the background stream constitutes a renewal process with interarrival time distribution f, the CDV affected cell stream will no longer be a renewal process. The complication arises from the fact that the time from the CBR arrival until the first background arrival depends on the interarrival time between the previous background arrival and the present CBR arrival However, if d is large, the number of background arrivals between two CBR arrivals will be large and since only the first interarrival interval has a distribution which differs from f, it will be an accurate approximation simply to use the equilibrium forward recurrence time independent of the previous background arrival.

Therefore, when the background process is a renewal process, when d is large, and when the load on the multiplex is 1, we may approximate the CDV affected cell stream with a renewal process with interarrival time distribution $1 + N_{\text{background}}(d(i-1), di)$. Here, it should be noticed that it is in general difficult to obtain a closed form expression for $N_{\text{background}}(s, t)$.

3 Queueing Analysis of a Superposition of CDV Affected Streams

In section 2, we derived expressions for the number of arrivals in a window from a single CDV affected CBR cell stream. In this section, we assume that a finite number of independent CDV affected CBR cell streams are multiplexed in the buffer. Due to the independence between different streams the distribution of the number of arrivals from the superposition can be found from a convolution of the individual distributions. The number of arrivals from a single stream is given in formula (7) and (9) in section 2. After convolution we can apply the Benes result for the virtual waiting time W_t in the G/D/1 queue. This result enables us to find the virtual waiting time distribution alone from knowledge on the number of arrivals in a window, both with windows starting at an arrivals and with arbitrary windows, see section 5.3 in [COST] for details. In the present case we shall apply the so-called "local load approximation", see section 5.3.2 in [COST] for details, which can be formulated as:

$$P\{W_t > r\} \cong \sum_{n=1}^{\infty} P\{N(t-n, t) = n+r\} - \rho \sum_{n=1}^{\infty} P\{N(t-n, t) = n+r \mid \text{an arrival at } t-n\} \qquad (12)$$

The result is only valid for stable queues i.e the load should be smaller than 1 and in this case it is normally easy to truncate the infinite summation in the numerical applications and still obtain accurate results.

3.1 Numerical results

To illustrate the approach we present two scenarios. In the first scenario three examples are considered: the superposition of 7, 8 and 9 CBR sources which have been exposed to CDV. These streams are multiplexed in a single multiplexing stage and after that the streams are offered to a FIFO with a load of 0.7, 0.8 and 0.9 which are corresponding to the superposition of 7, 8 and 9 streams, respectively, implying that the peak rate of the CBR sources are 0.1 corresponding to d = 10. The degree of CDV is controlled by the load on the CDV creating queues as described in section 2.

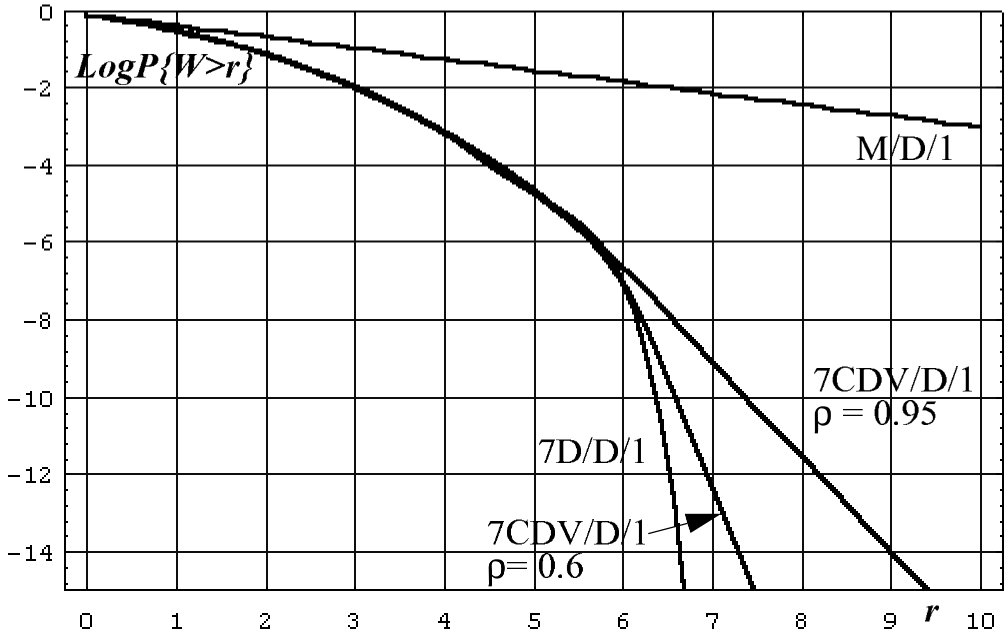

Figure 3 Comparison of the virtual waiting time distributions of 7CDV/D/1 queues (load = 0.7)

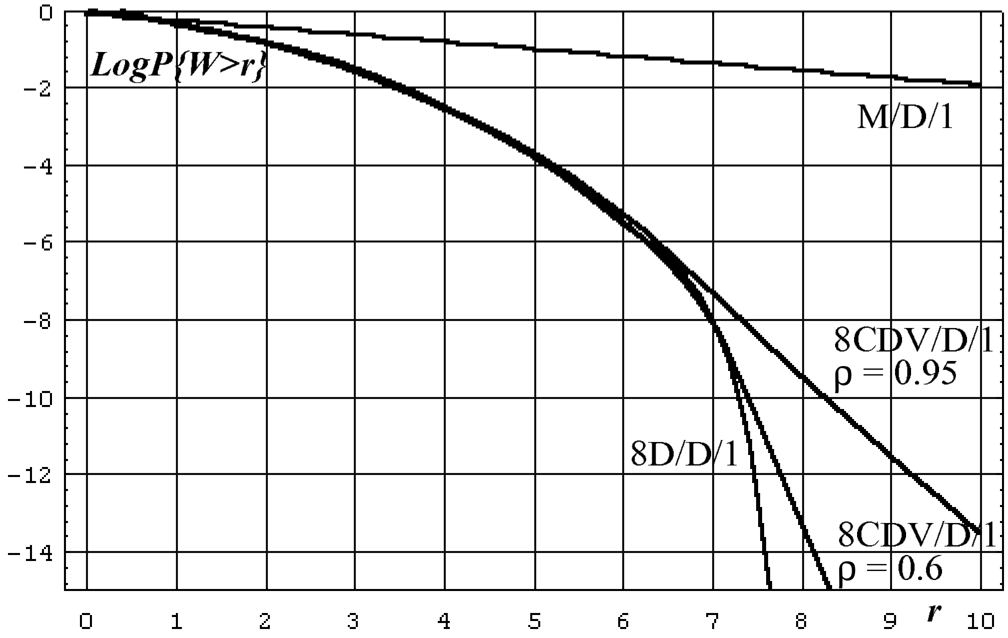

Figure 4 Comparison of the virtual waiting time distributions of 8CDV/D/1 queues (load = 0.8)

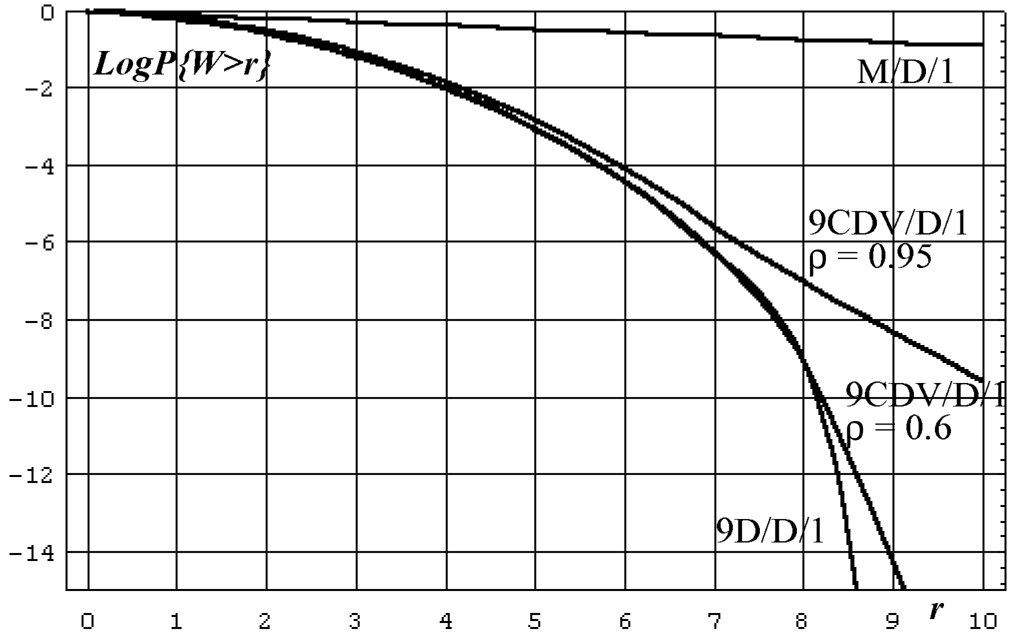

Figure 5 Comparison of the virtual waiting time distributions of 9CDV/D/1 queues (load = 0.9)

For a load of 0.6 in the CDV creating queues the queueing results differ very little from the result of the nD/D/1 queue while for a CDV creating load of 0.95 the difference become larger. However, the delay performance is still much better than for the M/D/1 case.

In the other scenario also three examples are considered: 14, 16 and 18 CDV affected CBR sources multiplexed in a FIFO with a load of 0.7, 0.8 and 0.9 respectively, implying d = 20.

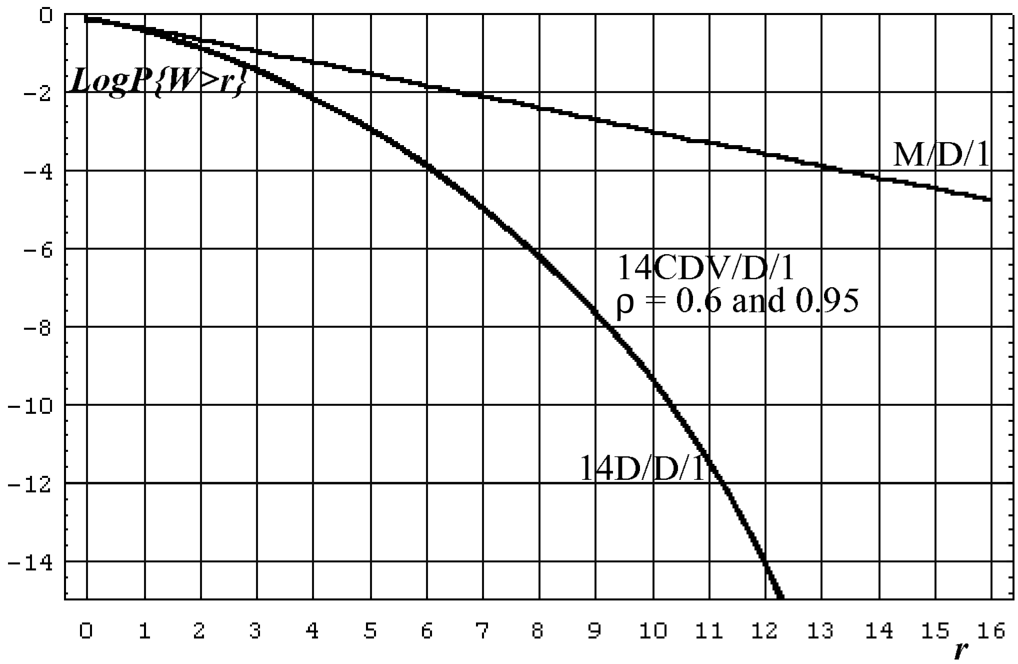

Figure 6 Comparison of the virtual waiting time distributions of 14CDV/D/1 queues (load = 0.7)

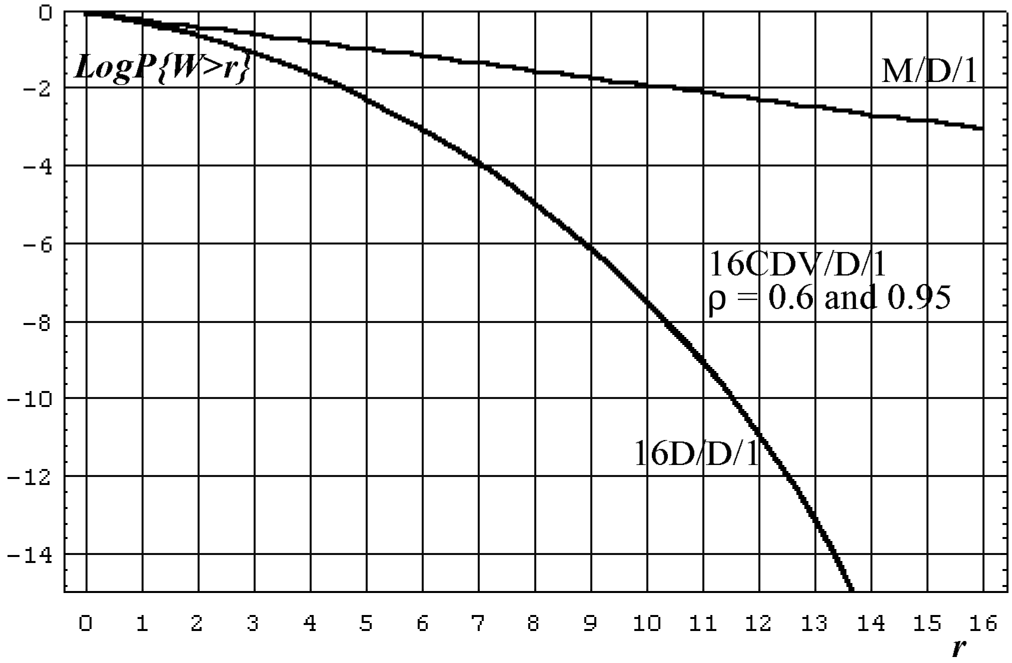

Figure 7 Comparison of the virtual waiting time distributions of 16CDV/D/1 queues (load = 0.8)

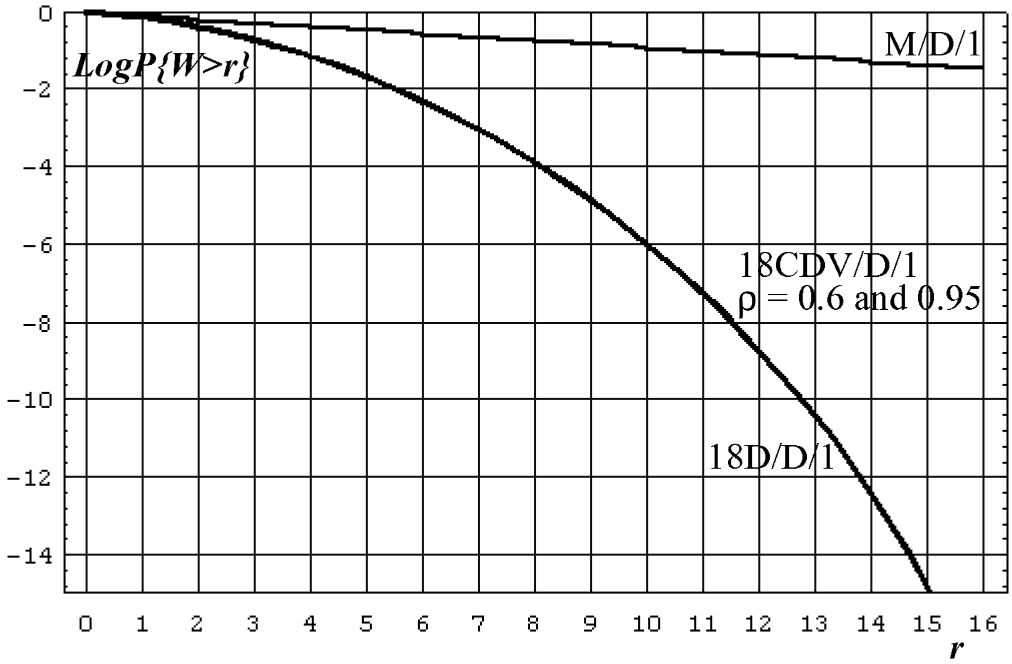

Figure 8 Comparison of the virtual waiting time distribution of 18CDV/D/1 queues (load = 0.9)

For these cases the results are practically identical to the results for the nD/D/1 queue even in the cases with a CDV creating load of 0.95.

An overall conclusion is that, *except for the case with multiplexing a very few high speed CBR cell streams, the effect of CDV in a single multiplexing queue is neglectable.*

4 CDV Characteristics after a series of multiplexing stages

The rather detailed diffusion model for characterizing the CDV after passage of a single queue becomes inappropriate when coming to the analysis of CDV after a series of queues. Instead a simpler model is needed. Here we shall give a brief description of the model presented in [BRe92], and apply the results to obtain the queueing performance of a number of CDV affected CBR cell streams.

Consider a discrete time queueing model consisting of M ATM queues in series. There are two types of arrivals to the system. First we have a reference connection entering the first queue and passing successively through all M queues. Secondly, for each queue k we have an interfering traffic entering queue k and immediately after service completion in queue k leaves the system. (See Figure 9.)

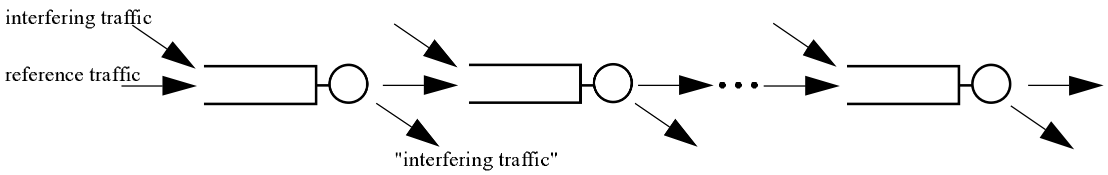

Figure 9 Queues in series with interfering traffic

Let the sequence $\{X_n^{k-1}\}_n$ for $k = 1, ..., M$ be the sequence of interarrival times of the reference traffic to queue k, that is X_n^{k-1} the number of slots (cells) between the arrival of reference cell no. n and $n+1$ to queue k.

The superposition of the interfering traffic is modelled as a batch Bernoulli process, that is the number of arrivals in successive times slots are independent identically distributed with common distribution f and generating function F. For simplicity it is assumed that the interfering traffic statistically is the same at each queue. Y_i^k denotes the number of arrivals in timeslot i at queue k.

The output process of the reference connection from queue k $\{X_n^k\}_n$ is a function of the input processes $\{X_n^{k-1}\}_n$ and $\{Y_n^k\}_n$. However, to arrive at a tractable model, the following heavy traffic assumption will be imposed:

Heavy Traffic Assumption:

At each point in time the k'th queue is non-empty.

Under this assumption all slots between two adjacent reference cell arrivals consists of interfering cells, implying that

$$X_n^k = \sum_{i=1}^{X_n^{k-1}+1} Y_i^k \tag{13}$$

where for $i = 1, \dots, X_n^{k-1}$ Y_i^k is the number of interfering arrivals in slot i and $Y^k_{X_n^{k-1}+1}$ is the number of interfering arrivals in the timeslot in which reference cell n arrived. To arrive at (13) we have also assumed that the reference connection has priority over the interfering cells. A similar expression would be valid if the interfering cells had priority over the reference connection, see [Bre92] for more details.

(13) shows that the process $\{X_n^k, k = 0,..,M\}$ is a branching process with immigration. Furthermore we get

$$P_k(s) = F(s) P_{k-1}(F(s)) \tag{14}$$

where $P_k(s)$ denotes the generating function of X_n^k.

From the general theory of branching processes it is possible to arrive at the following results:

Result 1: (Limit interarrival time distribution - infinitely many queues)

Assume that $E(Y_i^k) < 1$. When $k \to \infty$, then $X_n^k \to X_n^\infty$ in distribution where the generating function of X_n^∞ satisfy $P(s) = \prod_{j=1}^{\infty} F_j(s)$ in which $F_j(s)$ is recursively defined by $F_1(s) = F(s)$ and $F_j(s) = F(F_{j-1}(s))$.

Result 2: (Asymptotically a renewal process)

Let X_n^0 and X_{n+p}^0 be two possibly dependent interarrival times. Define:

$$(X_n^{k+1}, X_{n+p}^{k+1}) = \left(\sum_{i=1}^{X_n^k+1} Y_i^k, \sum_{i=1}^{X_{n+p}^k+1} Y_i^{k,p} \right) \tag{15}$$

where Y_i^k and $Y_i^{k,p}$ are independent of each other and satisfying the assumptions of result 1. Then

$$(X_n^k, X_{n+p}^k) \to (X_n^\infty, X_{n+p}^\infty) \tag{16}$$

in distribution, where $X_n^\infty, X_{n+p}^\infty$ are independent.

From these two results two important conclusions can be drawn:

1 When the reference cell stream has passed a sufficient number of queues the interarrival time distribution has converged towards a limit distribution which only depends on the interfering traffic and not on the original characteristics of the reference stream except the rate.

2 When the reference stream has passed a sufficient amount of queues it becomes a renewal stream.

Section 6 in [BRe92] argues by an interpolation argument between the light traffic case (no interfering traffic) and the heavy traffic case that the result 1 and 2 should also hold in moderate traffic with only the speed of the convergence changed.

We now take a closer look on how the squared coefficient of variation of the interarrival time changes as the cell stream passes through the network.

Assume that the reference connection constitutes a renewal process with rate 1 - p yielding $E(X_n^0) = \frac{p}{1-p}$ and arbitrary variance $Var(X_n^0)$.

From (14) we get

$$E(X_n^{k+1}) = E(Y_i)\,(1+E(X_n^k)) \text{ and } Var(X_n^{k+1}) = \frac{Var(Y_i)}{1-p} + p^2 Var(X_n^k) \tag{17}$$

From this equation it can be seen that the sequence $\{Var(X_n^k)\}_k$ is an increasing sequence if $Var(X_n^0) < \frac{Var(Y_i)}{(1-p)(1-p^2)}$ and it is decreasing if $Var(X_n^0) > \frac{Var(Y_i)}{(1-p)(1-p^2)}$ and in both cases the monotonic sequence converges towards the variance of X_n^∞.

To get an idea of how fast the convergence is we consider the case where the reference connection is a CBR connection with $Var(X_n^0) = 0$. Assume that the variance of the interfering stream $Var(Y_i) = p$ corresponding to Poisson traffic, and assume that the load on each of the queues are 1. From (17) it can be seen that

$$Var(X_n^k) = \frac{p}{1-p} \sum_{j=0}^{k-1} p^{2j} \tag{18}$$

For the limit case in which the number of queues to be passed is infinite and in the case with Poissonian interference, the CDV affected CBR cell stream will have a squared coefficient of variation

$$c^2 = \frac{Var(X_n^\infty)}{(E(X_n^\infty)+1)^2} = \frac{\frac{p}{(1-p)(1-p^2)}}{\frac{1}{(1-p)^2}} = \frac{p}{(1+p)} \tag{19}$$

where the $(E(X_n^\infty)+1)$ term is needed since $E(X_n^\infty)$ denotes the number of empty slots between two successive reference arrivals, and therefore the mean of the interarrival time is one larger than $E(X_n^\infty)$ An important implication of (19) is that *the squared coefficient of variation in the Poisson interfering case never exceeds 1/2.*

Consider the case in which the reference stream has interarrival time d = 10, corresponding to p = 0.9. Then the following table shows how the squared coefficient of variation of the reference stream varies as it passes through the queues of the network.

Table 1: The squared coefficient of variation as a function of the number of queues

M = 0	M = 1	M = 2	M = 4	M = 6	M = 10	M = 25
0	0.09	0.163	0.270	0.340	0.416	0.471

For M = 1, the squared coefficient of variation is $\frac{p/(1-p)}{d^2} = \frac{(1-1/d)/(1/d)}{d^2} = \frac{d-1}{d^2}$ which is exactly the squared coefficient of variation of the CDV interarrival time distribution given in section 2.2.

The conclusions that a limit interarrival time distribution emerges, and that it depends only on the characteristics of the interfering stream together with the load of the reference stream also appears in the model of [AMe93]. In this model, which does not assume heavy load, the same scenario with a series of queue with two types of arrivals are investigated. Here the reference cell stream is a discrete time renewal process with a truncated, but else, general interarrival time distribution. The interfering streams are Poisson streams or superpositions of Bernoulli streams. Each queue is treated independently and the interdeparture interarrival time of the reference cell stream at queue k is computed numerically and the result is used as (reference stream) input to queue $k + 1$.

4.1 Numerical results

As in the previous case we consider two scenarios. In the first scenario 7, 8 and 9 CDV affected cell streams are multiplexed corresponding to the load of 0.7, 0.8 and 0.9 respectively. In the second scenario we investigate the superposition of 14, 16 and 18 CDV affected cell streams also corresponding to the load of 0.7, 0.8 and 0.9 respectively. From the values given in the table we approximate each CDV affected stream by renewal streams with an Erlang-n interarrival time distribution. We have taken n = 11 (corresponding to passage of a single queue), n = 4 (corresponding to passage of 4 queues), and n = 2 (corresponding to an infinite number of queues). As in section 3, we have used the local load approximation of the Benes result for finding the virtual waiting time distribution.

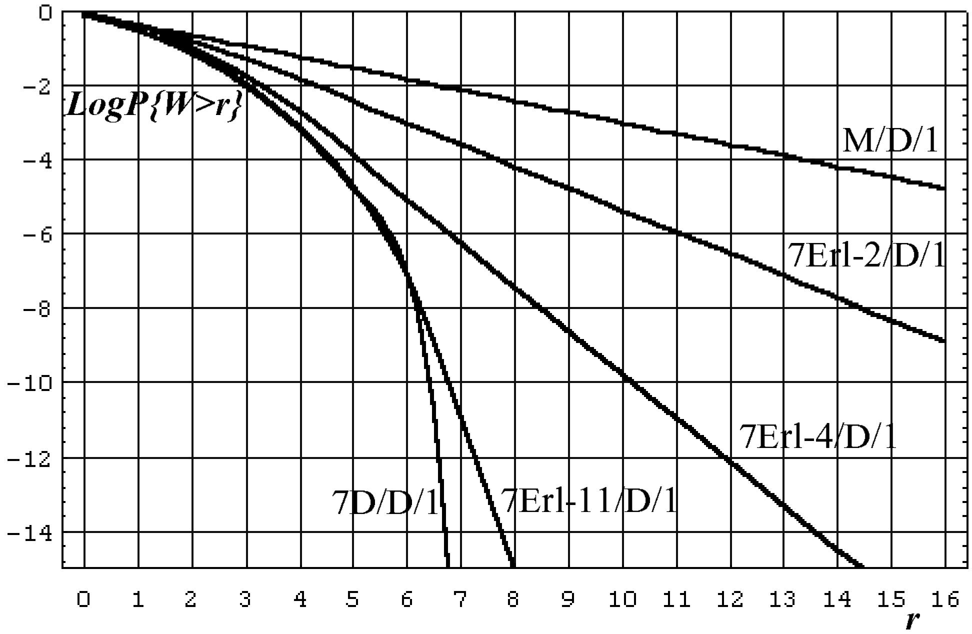

Figure 10 Comparison of the virtual waiting time distributions of 7Erl-*n*/D/1 queues (load = 0.7)

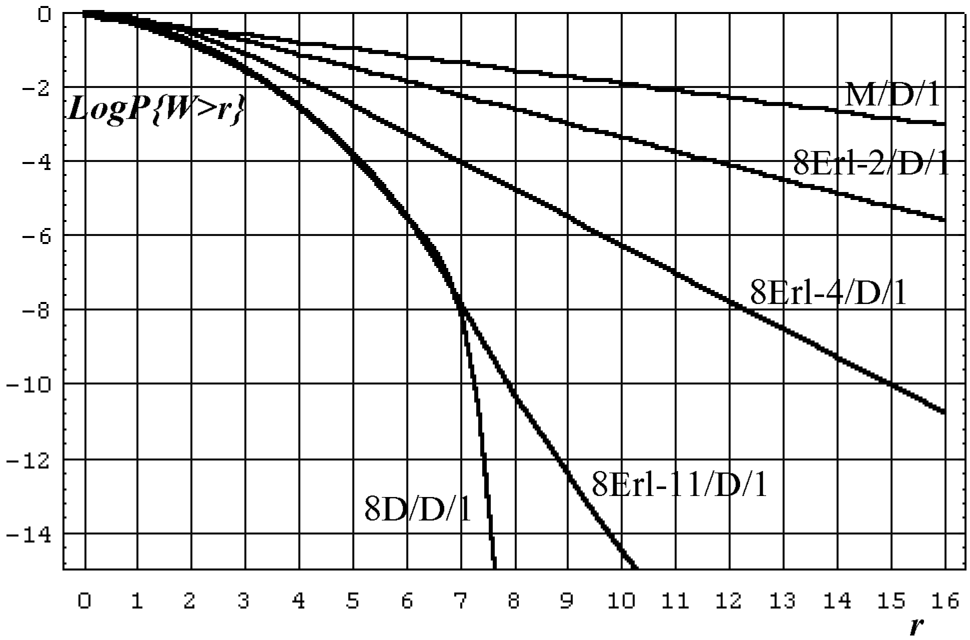

Figure 11 Comparison of the virtual waiting time distributions of 8Erl-*n*/D/1 queues (load = 0.8)

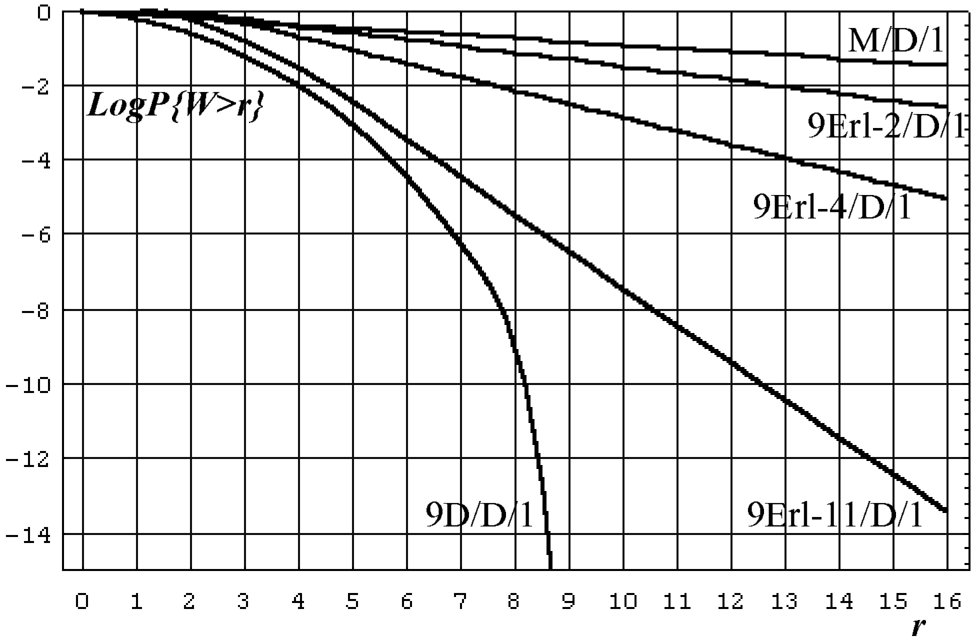

Figure 12 Comparison of the virtual waiting time distributions of 9Erl-*n*/D/1 queues (load = 0.9)

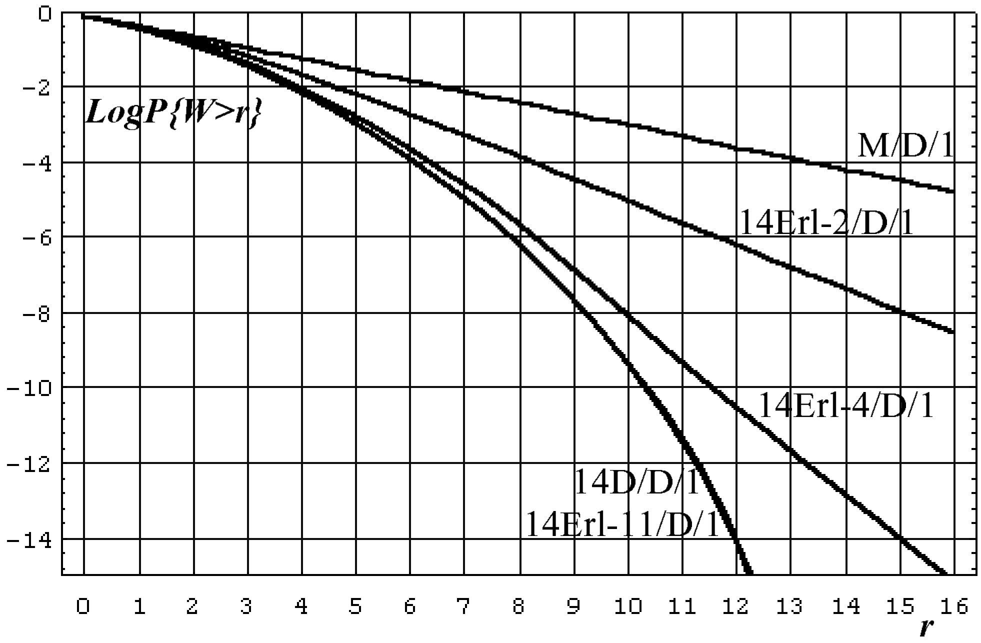

Figure 13 Comparison of the virtual waiting time distributions of 14Erl-*n*/D/1 queues (load = 0.7)

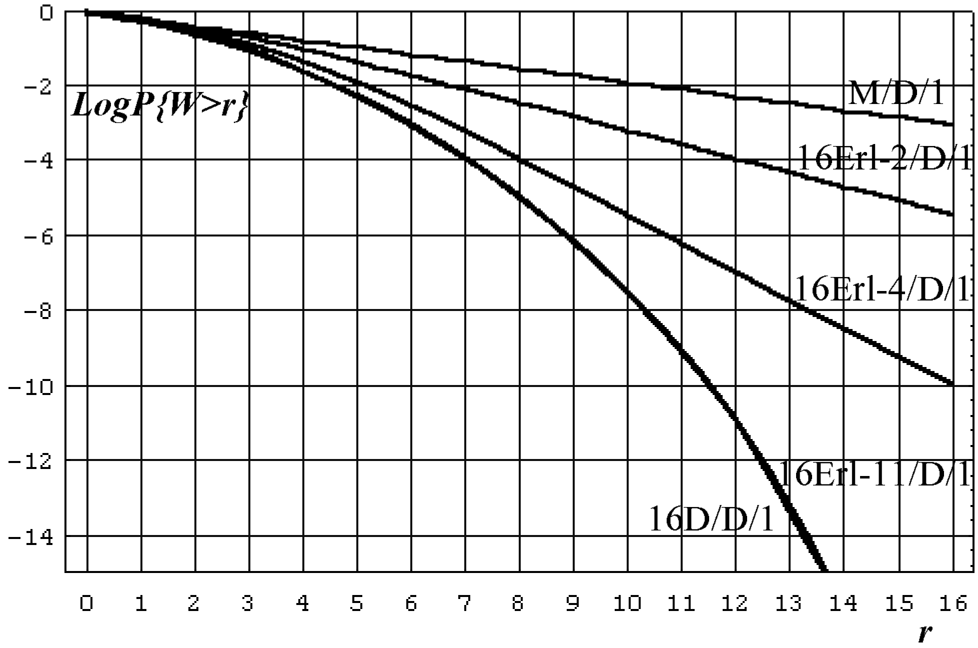

Figure 14 Comparison of the virtual waiting time distributions of 16Erl-*n*/D/1 queues (load = 0.8)

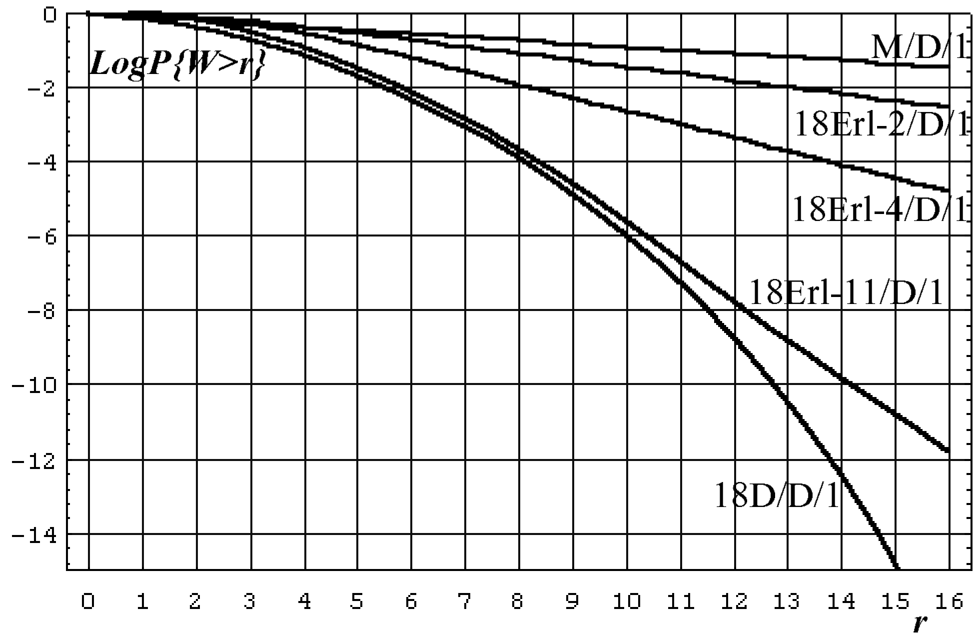

Figure 15 Comparison of the virtual waiting time distributions of 18Erl-*n*/D/1 queues (load = 0.9)

As the result shows then the affect of CDV comes more and more apparent as the number of queues which the CBR streams has passed increases. However, an important result of the model described in section 4 is that as long as we have Poissonian interference then the delay performance of the multiplexing of a number of CDV affected CBR cell streams will not be worse than the superposition of the same number of renewal processes with squared coefficients of variation 1/2.

5 Conclusion

In this paper we have investigated the effect of cell delay variation on the queueing performance when a queue is offered a number of CDV affected CBR cell streams. From the first and rather detailed model we conclude that the degree of CDV created in a single FIFO multiplexing stage has practically no influence on queueing performance at the next stage. From the second model which assumed heavy load we conclude that in case of Poissonian interfering traffic the squared coefficient of variation of the individual cell streams will not exceed 1/2 implying that dimensioning based on the simple M/D/1 model will be conservative.

References

[AMe93] M. D'Ambrosio, R. Melen, *"On the Modification of the Cell Streams Within an ATM Network"*, COST 242 TD(93) 005

[BRe92] J. L. v. Berg, J. A. C. Resing, *"The Change of Traffic Characteristics in ATM Networks"*, COST 242 TD(92)040

[Bla93] S. Blaabjerg, *"Cell Delay Variation in a FIFO queue: A Diffusion Approach"*,. High Speed Networks, Raleigh, October 1993.

[BMS93] C.C. Bisdikian, W.Matragi, K. Sohraby, *"A Study of the Jitter in ATM Multiplexers"*, High Speed Networks, IFIP, Raleigh, 1993.

[BSi87] P. Brown, A. Simonian, *"Pertubation of a Periodic Flow in a Synchronous Server"*, Performance 87.

[CCITT] CCITT SG XVIII Recommendation I.371.

[CLe66] D.R. Cox, P.A.W. Lewis, *"The Statistical Analysis of Series of Events"*, Methuen 1966.

[COST] COST 224, *"Performance Evaluation and Design of Multi Service Networks"*, Commission of the European Communities, October 1991.

[CTF92] P. Castelli, A. Tonietti, A. Forcina, *"Dimensioning Criteria for Policing Functions ATM Networks"*, INFOCOM'92, Session 6A.

[DDJR91] B. Doshi, S. Dravila, P. Johri, G. Ramamurthy, *"Memory, Bandwidth, Processing and Fairness Considerations in Real Time Congestion Control for Broadband Networks"*, ITC-13, 1991, pp. 153-159.

[GBDR92] F. Guillemin, P. Boyer, A. Dupuis, L. Romoeuf, *"Peak Rate Enforcement in ATM Networks"*, INFOCOM'92, Session 6A.

[GCa92] J.Garcia, O. Casals, *"Approximate Analysis of Statistical Multiplexing of Variable Bit Rate and Periodic Sources"*, COST 242 TD(92) 042.

[GRo91] F. Guillemin, J. W. Roberts, *"Jitter and Bandwidth Enforcement"*, Globecom'91, Session 9.3.

[Hue94] F. Huebner, *"Dimensioning of a Peak Cell Rate Monitor Algorithm Using Discrete-Time Analysis"*, ITC-14, France 1994.

[Kel93] F.P. Kelly, *"Mathematical Models of MultiService Networks"*, IMA conference on Complex Stochastic Systems and Engineering, Leeds, September 1993.

[Kle76] L. Kleinrock, *"Queueing Systems."* Volume II, Wiley Interscience.

[Mat93] M. Mateescu, *"On Allocation Schemes for the Interconnection of LANs and Multimedia Sources over Broadband Networks"*, Proceedings of the First International Conference on Local Area Network Interconnection, North-Carolina, Oct. 20-22, 1993, pp. 47-64.

[Skl94] A. Skliros, *"A Connection Admission Algorithm for ATM Traffic Distorted by Cell Delay Variation"*, ITC-94, France 1994.