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NETWORK DIMENSIONING BASED ON REFINED BLOCKING MEASURES

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1 Introduction

In this paper extensions of the network dimensioning algorithms are presented for handling cases where the Poisson arrival process and the exponential holding time assumptions can be relaxed. The main purpose of this extension is to improve the network dimensioning algorithms to fulfil the expected nature of the future ATM, where the call arrival process will differ significantly from the Poisson process and the holding time distribution will be deviate from the exponential distribution. In order to fulfil these requirements we have to investigate more general traffic models [1].

For dimensioning purposes the traffic offered to each route is characterized by the mean and the variance (or the peakedness defined by the ratio of variance and mean). Based on these two parameters four blocking measures are proposed to compute link blocking probabilities. Two measures are based on the BPP approximation, and compute the traffic and call congestion. The third blocking measure is a traffic congestion based on the truncated normal approximation, and the fourth one is the Hayward approximation. The principle behind the measures is written in [1].

To upgrade the dimensioning algorithms (Equivalent Link Blocking (ELB)[2], Iterated Equivalent Link Blocking (IELB)[3] and Fixpoint method (FIXPOINT)[4]) only the replacement of the Erlang formula to the suggested blocking measures is needed. In case of the ELB and IELB the capacities have to be calculated from the blocking measures. In the FIXPOINT algorithm the blocking values have to be computed using the measures.

2 Notation and Data

M: Mean of the occupancy distribution to be approximated

V: Variance of the occupancy distribution to be approximated

Z: Peakedness of occupancy distribution (defined by V/M)

α, β: Parameters of occupancy distribution

$f(x)$: Truncated Normal density function

$\Phi(x)$: Normal distribution

$\phi(x)$: Normal density function

μ: service rate

p_i: Pr{i number of servers are occupied} (link occupancy)

C: Capacity of link (number of servers)

ρ: Mean of offered traffic

σ^2: Variance of offered traffic

B1, B2, B3, B4: Blocking measures

3 Upgrading the ELB and IELB methods

In the ELB method [2] the Erlang formula on pp. 4 has to be replaced by Bx with input parameters: $B_j^0 = Bx\left(\rho_j^0, \sigma_j^2, C_j\right)$, ($x$ can be 1, 2, 3 or 4), where the added parameter σ_j^2 is computed as

$$\sigma_j^2 = \sum_{r \in R_j} A_{jr}^2 \sigma_r^2 \prod_{i \neq j} (1 - B_i^0)^{A_{ir}} \quad .$$

In case of IELB the same replacement can be performed in the algorithm (3.6 section on pp. 4 in the document of [3]).

4 Upgrading the FIXPOINT method

In the FIXPOINT procedure [4] the Erlang formula in step 3 has to be replaced by Bx (x can be 1, 2, 3 or 4) with parameters

$$B_j = Bx\left((1 - B_j)^{-1} \sum_r A_{jr} \nu_r \prod_i (1 - B_i)^{A_{ir}},\ (1 - B_j)^{-1} \sum_r A_{jr}^2 \sigma_r^2 \prod_i (1 - B_i)^{A_{ir}},\ \tilde{C}_j \right)$$

5 Blocking measure based upon the BPP approximation (B1)

The formula $B1\,(M, V, C)$ can be computed by the next algorithm:

5.1 Step 1: Computing parameters of BPP

Let the mean holding time be 1 (μ=1).

$$\beta = 1 - \frac{1}{Z} \qquad \alpha = M(1 - \beta)$$

If $\beta < 0$ then $\beta = \beta'$, which is the closest to the computed β such that $N = -\frac{\alpha}{\beta'}$ must be integer.

5.2 Step 2: Computing the truncated BPP distribution

If $\beta = 0$ then $p_0 = e^{-\alpha}$.

If $\beta \neq 0$ then $p_0 = (1 - \beta)^{\frac{\alpha}{\beta}}$.

For $i > 0$: $p_i = p_{i-1} \dfrac{\alpha + (i-1)\beta}{i}$

If $\beta < 0$ and $i > N$ then $p_i = 0$

The truncated distribution: $p_i^{tr} = \dfrac{p_i}{\sum_{j=0}^{C} p_j}$

If $\beta < 0$ and $C > N$ then $p_i^{tr} = p_i$

5.3 Step 3: Traffic Congestion computation

$$B1 = \frac{M - M_{tr}}{M}, \text{ where } M_{tr} = \sum_{j=0}^{C} j p^{tr}_j.$$

6 Blocking measure based upon the BPP approximation (B2)

The calculating of this measure can be performed by the same procedure as in case of B1 with the exception of the last step:

Step 3: Call Congestion computation:

$$B2 = \left(1 + \frac{C}{M}(Z-1)\right) p_C^{tr}$$

7 Blocking measure based upon the truncated normal distribution (B3)

The formula $B3(M, V, C)$ can be computed by the following algorithm:

7.1 Step 1: Computing parameters of the truncated normal distribution

Define $x = \dfrac{\alpha}{\sqrt{2\beta}}$, and $G(x) = \dfrac{\varphi(x)}{\Phi(x)}$.

Solve the equation of $\dfrac{x^2 + 1 - xG(-x)}{(G(-x) - x)^2} = \dfrac{V + M^2}{M^2}$, with x unknown.

It can be seen that the solution is unique in case of $0 < Z < M$, see [1], which is related to the $1 < \frac{V + M^2}{M^2} < 2$ case.

Let us denote the solution by *y*.

The parameters of truncated normal distribution can be calculated as

$$\beta = \frac{[G(-y) - y]^2}{2M^2}$$

$$\alpha = y\sqrt{2\beta}$$

7.2 Step 2: Computing the carried traffic of the truncated normal distribution

From the normal distribution $f(x) = \begin{cases} ge^{-\alpha x}e^{-\beta x^2} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$ the carried traffic can be computed after truncation at *C*:

$$M_{tr} = \frac{\frac{-\alpha}{2\beta}\left(\Phi\left(\frac{-\alpha}{\sqrt{2\beta}}\right) - \Phi\left(-\sqrt{2\beta}C - \frac{\alpha}{\sqrt{2\beta}}\right)\right) + \frac{1}{\sqrt{2\beta}}\left(\varphi\left(\frac{\alpha}{\sqrt{2\beta}}\right) - \varphi\left(\sqrt{2\beta}C + \frac{\alpha}{\sqrt{2\beta}}\right)\right)}{\Phi\left(\sqrt{2\beta}C + \frac{\alpha}{\sqrt{2\beta}}\right) - \Phi\left(\frac{\alpha}{\sqrt{2\beta}}\right)}$$

7.3 Step 3: Traffic Congestion computation

$$B3 = \frac{M - M_{tr}}{M}$$

8 Blocking measure based upon the Hayward approximation (B4)

The Hayward approximation can be calculated as $B4 = E\left(\frac{M}{Z}, \frac{C}{Z}\right)$, where E(.,.) is the Erlang formula: $E(\rho, C) = \frac{\frac{\rho^C}{C!}}{\sum_{j=0}^{C} \frac{\rho^j}{j!}}$

References

[1] S. Molnár, S. Blaabjerg, "On some simple blocking approximations for the G/G/c queue", COST 242 TD(94)001

[2] A. Faragó, "A Simplified Dimensioning Model Based on Equivalent Link Blocking Probabilities", Updated Version, March 1994

[3] S. Blaabjerg, "Optimization using Equivalent Link Blockings", Corrected Version, Nov. 1993

[4] A. Faragó et. al. "Resource separation - an Efficient Tool for Optimizing ATM Network Configuration", NETWORKS'94, Sept. 1994